

# Exam 2 Solution

## Problem 1 - (40 points)

Use:  $V_t = 1V$   
 $k_n' (W/L) = 2mA/V^2$   
 $\lambda = 0$

For DC analysis, assume that the capacitors act as an open.  
 The current source is not ideal and has a voltage drop across it.

(a) Solve for the DC currents:

- $I_1 = 0A$
- $I_{S2} = 2.97mA$

(b) Solve for the DC voltages:

- $V_{G2} = 7V$
- $V_{S2} = 4.24V$
- $V_{S1} = 3V$

(c) Verify that transistor M1 is saturated.

(d) State the DC bias point for transistor M2.

(e) Assuming that the transistor amplification is  $V_{S2}/V_{IN} = -4V/V$ . Assume the input frequency is operating within the circuits operating range. Assume that the amplification does not pull the transistors out of saturation. Will  $V_{S2}$  ever be 3V? Why or why not?

Parts (a) and (b): Assume saturation mode for  $M_1, M_2$ . Open up capacitors.

$I_1 = 0A$  (no current through gate)

$M_1$  transistor:  $V_{G1} = 5V$  (no current through gate)

$I_{S1} = 1mA$

$I_{S1} = \frac{1}{2} k_n' \left(\frac{W}{L}\right) (V_{GS1} - V_t)^2$

$\therefore 1mA = \frac{1}{2} (2mA) (V_{GS1} - 1)^2$

$\therefore 1 = V_{GS1}^2 - 2V_{GS1} + 1$

$\therefore V_{GS1}^2 - 2V_{GS1} = 0 = V_{GS1}(V_{GS1} - 2) = 0$

$\rightarrow V_{GS1} = 0V, 2V$

Need  $V_{GS1} > V_t$ , so  $V_{GS1} = 2V$ .

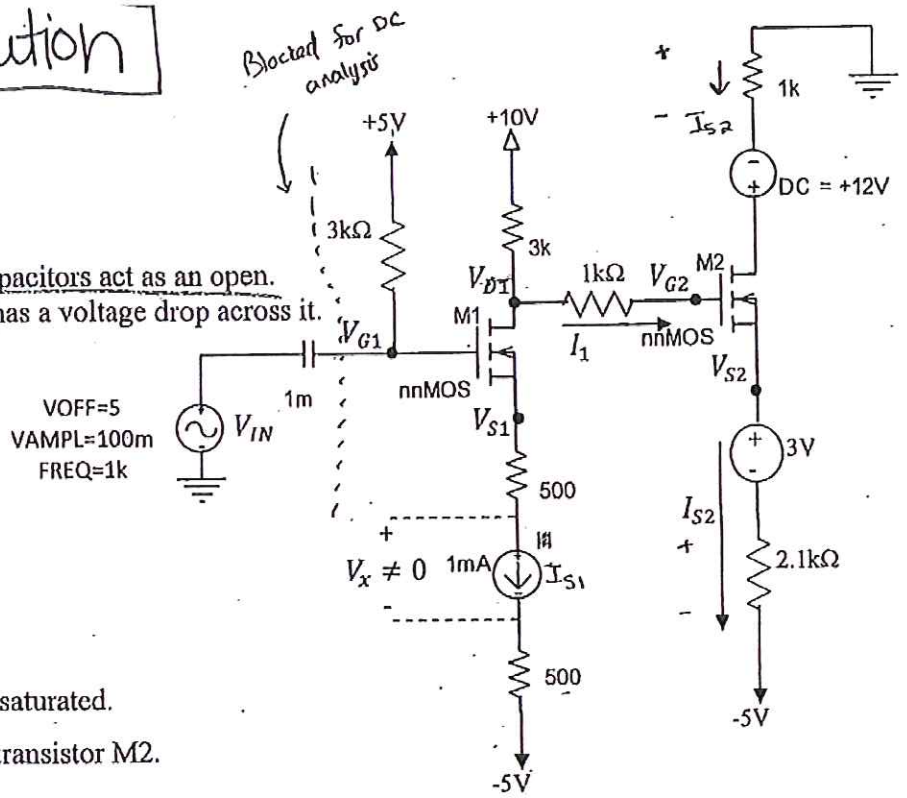
$V_{GS1} = V_{G1} - V_{S1}$ , so  $V_{S1} = V_{G1} - V_{GS1} = 5V - 2V = 3V = V_{S1}$ .

$V_{D1} = 10V - (1mA \cdot 3k\Omega) = 10V - 3V = 7V$

Need  $V_{D1} > V_{G1} - V_t$ :  $(V_{D1} = 7V) > (V_{G1} - V_t = 5V - 1V = 4V)$ .  $\checkmark$

$\therefore M_1$  is saturated

$V_{D1} = V_{G2} = 7V$  (no current through gate)



M2 transistor:  $V_{GS2} = V_{DS1} = 7V$

$$V_{S2} = -5V + I_{S2}(2.1k) + 3V = -2V + I_{S2}(2.1k)$$

$$I_{S2} = \frac{1}{2} k_n' \left(\frac{W}{L}\right) (V_{GS2} - V_t)^2 = \frac{1}{2} k_n' \left(\frac{W}{L}\right) (V_{GS2} - V_{S2} - V_t)^2$$

$$\therefore I_{S2} = \frac{1}{2} k_n' \left(\frac{W}{L}\right) (7 - (-2 + I_{S2}(2.1k)) - 1)^2$$

$$I_{S2} = \frac{1}{2} k_n' \left(\frac{W}{L}\right) (8 - I_{S2}(2.1k))^2$$

$$I_{S2} = \frac{1}{2} (2m) (64 - 2(I_{S2} \cdot 16.8k) + I_{S2}^2 (2.1k)^2)$$

$$I_{S2} = (1m) (64 - 33.6k I_{S2} + 4.41M I_{S2}^2)$$

$$I_{S2} = 64m - 33.6 I_{S2} + 4.41k I_{S2}^2$$

$$0 = 64m - 34.6 I_{S2} + 4.41k I_{S2}^2$$

$$I_{S2} = \frac{34.6 \pm \sqrt{(34.6)^2 - 4(4.41k)(64m)}}{2(4.41k)} = \frac{34.6 \pm \sqrt{68.2}}{8.82k}$$

$$= 4.86mA, 2.97mA$$

Try  $I_{S2} = 2.97mA$ .  $V_{DS2} = 0V - (2.97mA \cdot 1k) + 12V = 9.03V$

$$V_{S2} = -2V + (2.97mA \cdot 2.1k) = 4.24V$$

Need  $V_{GS2} > V_t$ :  $(V_{GS2} = V_{GS2} - V_{S2} = 7V - 4.24V = 2.76V) > (V_t = 1V)$  ✓

Need  $V_{DS2} > V_{GS2} - V_t$ :  $(V_{DS2} = 9.03V) > (V_{GS2} - V_t = 7V - 1V = 6V)$ . ✓

$\therefore$  saturated

$$I_{S2} = 2.97mA, V_{S2} = 4.24V$$

Part (c): Need  $V_{GS1} > V_t$ :  $(V_{GS1} = 2V) > (V_t = 1V)$ . ✓

Need  $V_{DS1} > V_{GS1} - V_t$ :  $(V_{DS1} = 7V) > (5V - 1V = 4V)$ . ✓  $\therefore$  Saturated

Part (d): DC bias point for M2 is  $V_{GS2} = V_{GS2} - V_{S2} = 7V - 4.24V = 2.76V$   
or  $I_{S2} = 2.97mA$

Part (e):  $V_{S2} = V_{S2DC} + V_{S2AC}$

$$V_{S2DC} = 4.24V$$

$$V_{S2AC} = -4\frac{V}{V} \cdot 100mV \sin(\omega t) = -400mV \sin(\omega t)$$

$$V_{S2} = 4.24V - 0.4V \sin(\omega t)$$

Peak values of  $V_{S2}$  are  $4.24V + 0.4V = 4.64V$   
 $4.24V - 0.4V = 3.84V$

So  $V_{S2}$  will never be 3V.

**Problem 1 – (40 points)**

Solution

Use:  $V_t = 1V$   
 $k_n'(W/L) = 1mA/V^2$   
 $\lambda = 0$   
 $V_{IN} = (5 + 10\text{msin}(20t))V$

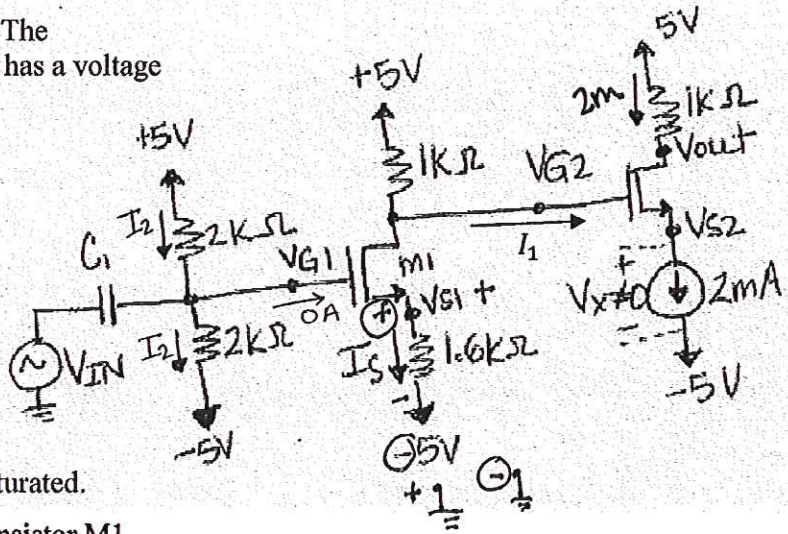
For DC analysis, assume that the capacitors act as an open. The current source is not ideal and has a voltage drop across it.

(a) Solve for the DC currents:

- a.  $I_1 = 0$
- b.  $I_S = 1.4\text{mA}$

(b) Solve for the DC voltages:

- a.  $V_{G2} = 3.6V$
- b.  $V_{S2} = 0.6V$
- c.  $V_{S1} = -2.7V$



(c) Verify that transistor M2 is saturated.

(d) State the DC bias point for transistor M1.

(e) Assuming that the transistor amplification is  $V_{out}/V_{IN} = +5V/V$ . Assume the input frequency is operating within the circuits operating range. Assume that the amplification does not pull the transistors out of saturation. Draw a rough sketch of the total instantaneous value seen at  $V_{out}$  using the  $V_{IN}$  value stated above.

a)  $I_1 = 0$

b) 
$$I_S = \frac{1}{2} k_n' \left(\frac{W}{L}\right) (V_{GS1} - V_t)^2 = \frac{1}{2} (1\text{mA}) (V_{G1} - V_{S1} - 1)^2$$

$V_{G1} = 0$  OR  $+5 - I_2(2k) - I_2(2k) + 5 = 0$

$I_2 = \frac{10}{4k} = 2.5\text{mA}$

$-5 + I_2(2k) = V_{G1} = 0$

$V_{S1} = -5 + I_S(1.6k)$

$$\frac{2 I_S}{1\text{mA}} = (0 + 5 - I_S(1.6k) - 1)^2 = (4 - I_S(1.6k))^2 = 16 - 12800 I_S + I_S^2(1.6k)^2$$

$0 = I_S^2(1.6k)^2 - 14,800 I_S + 16$

$$I_S = \frac{14,800 \pm \sqrt{14,800^2 - 4(16)(1.6k)^2}}{2(1.6k)^2} = 4.3\text{mA and } 1.4\text{mA}$$



If  $I_s = 4.3\text{mA}$  then  $V_{s1} = -5 + I_s(1.6\text{k}) = +1.88$   
 $V_{GS1} = 0 - 1.88 = -1.88 < V_t \times \underline{\text{NO}}$

$I_s = 1.4\text{mA}$

$V_{s1} = -5 + (1.4\text{mA})(1.6\text{k}) = -2.7\text{V} = V_{s1}$

$\therefore V_{GS1} = 2.7\text{V} > V_t \therefore \text{ON}$

$V_{D1} = V_{G2} = 5 - 1\text{k}(1.4\text{mA}) = 3.6\text{V} = V_{G2}$

$3.6 = V_{D1} - V_{s1} > V_{G1} - V_{s1} - V_t = 0 - 1 \checkmark \text{ TRUE, SAT}$

$I_{D2} = I_{S2} = 2\text{mA} = \frac{1}{2}(1\text{mA})(3.6 - V_{s2} - 1)^2$

$4 = (2.6 - V_{s2})^2$

$\pm\sqrt{4} = 2.6 - V_{s2}$

$\therefore V_{s2} = 2.6 \pm 2 = 4.6 \text{ OR } 0.6$

If  $V_{s2} = 4.6$  then  $V_{GS2} = 3.6 - 4.6 = -1 < V_t = 1 \therefore \text{NOT ON!}$

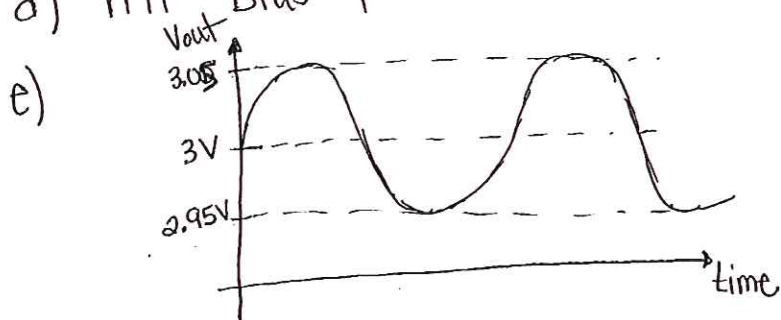
$V_{s2} = 0.6\text{V}$

c)  $V_{GS2} = 3.6 - 0.6 = 3\text{V} > V_t = 1 \therefore \text{ON}$

$V_{D2} = 5 - 1\text{k}(2\text{mA}) = 3\text{V}$

$3\text{V} = V_{D2} - V_{s2} > V_{G2} - V_{s2} - V_t = 2.6\text{V} \checkmark \text{ SATURATED}$

d) ml bias point is  $I_D = 1.4\text{mA}$  OR  $V_{GS1} = 2.7\text{V}$



$V_{out, total} = V_{out, DC} + V_{out, AC}$   
 $V_{out, total} = 3 + 5(10\text{ms}\sin(20\pi t))$

NAME: \_\_\_\_\_

ECE2280

Quiz #3

(open book/notes)

I certify that the work below is my own.

Signature: \_\_\_\_\_

Let  $V_t=1V$ ,  $k_n'(W/L)=1mA/V^2$ ,  $v_{sig}$  is an AC source,  $\lambda=0$  (for all transistors) and assume all transistors are saturate  
 Transistor 1 has DC values:  $V_{GS}=3V$ ,  $I_D=2mA$ .  
 Transistor 2 has DC values:  $V_{GS}=5V$ ,  $I_D=8mA$ .

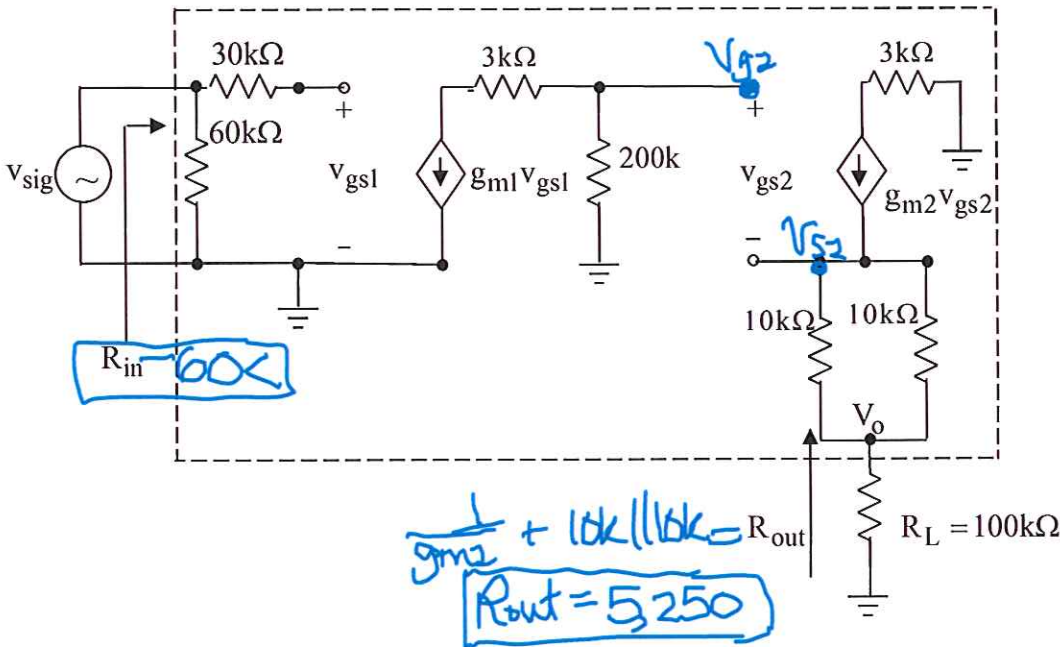
$$g_{m1} = (m)(3-1) = 2m$$

$$g_{m1} = \sqrt{2(1m)(2m)} = 2m$$

$$g_{m2} = (1m)(5-1) = 4m$$

For the following hybrid- $\pi$  equivalent circuit, find the following values:

- (a)  $R_{in}$  (input resistance—ignore the input source,  $V_{sig}$ )
- (b)  $R_{out}$  (output resistance—ignore  $R_L$  {no load is connected})
- (c) ideal midband gain,  $\frac{V_o}{V_{sig}}$



$$\frac{1}{g_{m2}} + 10k \parallel 10k = R_{out}$$

$$R_{out} = 5.25k$$

$$V_o = g_{m2} V_{gs2} (100k)$$

$$V_{gs2} - V_{g2} \quad V_{s2} = -g_{m1} V_{gs1} (200k) - g_{m2} V_{gs2} (105k)$$

$$V_{gs2} (+g_{m2} (105k)) = -g_{m1} V_{gs1} (200k)$$

$$V_{gs2} = \frac{g_{m1} V_{gs1} (200k)}{(1 + g_{m2} (105k))}$$

$$V_{gs1} = V_{sig} \quad \left[ \frac{-g_{m1} V_{sig} (200k)}{(1 + g_{m2} (105k))} \right] \Rightarrow \frac{V_o}{V_{sig}} = -380 \frac{V}{V}$$

**Problem 2 – (35 points)**

Use:  $V_t = 1V$

$$k_n'(W/L) = 2mA/V^2$$

$V_{sig}$  is an AC source

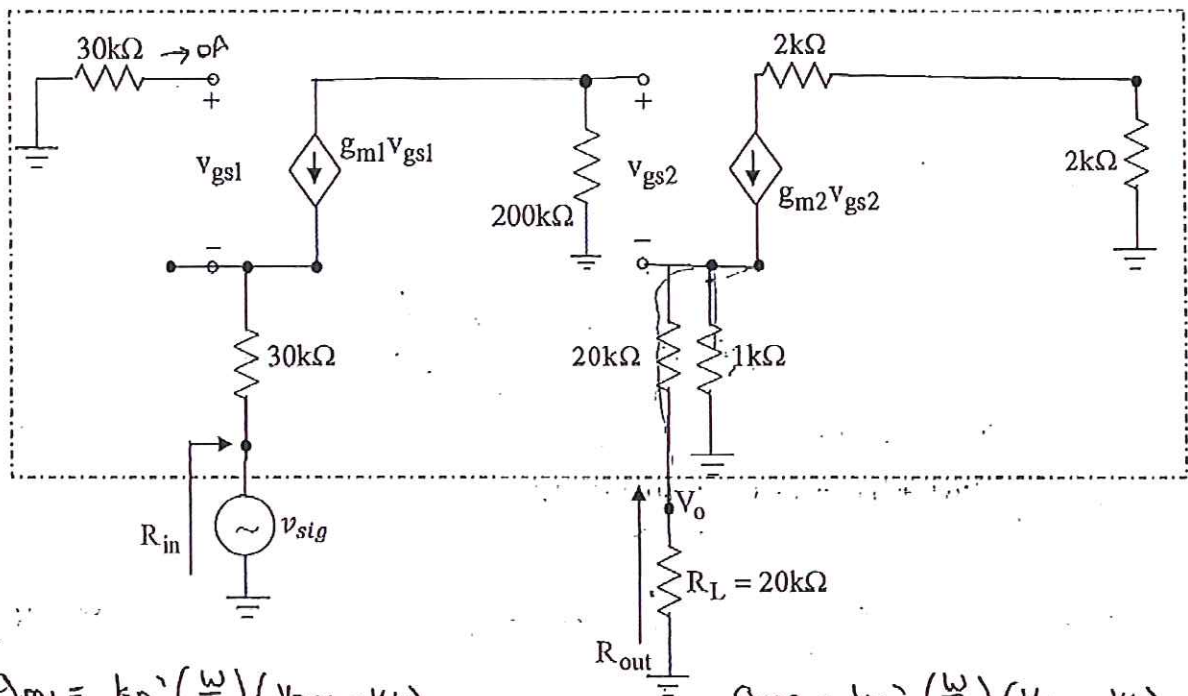
Transistor 1 has DC values:  $V_{GS} = 1.5V, I_D = 0.25mA$

Transistor 2 has DC values:  $V_{GS} = 13.5V, I_D = 156.25mA$

$\lambda = 0$  (for all transistors) and assume all transistors are saturated

For the following hybrid- $\pi$  equivalent circuit, find the following values:

- (a)  $R_{in}$  (input resistance – ignore the input source,  $v_{sig}$ )  
 (b)  $R_{out}$  (output resistance – ignore the load resistor,  $R_L$ )  
 (c) ideal overall midband gain,  $\frac{V_o}{v_{sig}}$ .



$$g_{m1} = k_n' \left( \frac{W}{L} \right) (V_{GS1} - V_t)$$

$$= (2m)(1.5 - 1) = (2m)(0.5) = 1m$$

$$g_{m2} = k_n' \left( \frac{W}{L} \right) (V_{GS2} - V_t)$$

$$= (2m)(13.5 - 1)$$

$$= (2m)(12.5) = 25m$$

a)  $R_{in} = 30k\Omega + \frac{1}{g_{m1}} = 30k\Omega + \frac{1}{1m} = 30k\Omega + 1k\Omega = \boxed{31k\Omega}$

b)  $R_{out} = 20k\Omega + 1k\Omega \parallel \frac{1}{g_{m2}} = 20k\Omega + \frac{1}{\frac{1}{1k\Omega} + g_{m2}} = 20k\Omega + \frac{1}{1m + 25m}$

$$= 20k\Omega + \frac{1}{26m} = \boxed{20.038k\Omega}$$



$$c) V_o = 0V + \left( \frac{g_{m2} V_{gs2} \cdot 1k\Omega}{1k\Omega + 20k\Omega + 20k\Omega} \right) \cdot 20k\Omega = \frac{g_{m2} V_{gs2}}{41} \cdot 20k\Omega = \frac{g_{m2} V_{gs2} \cdot 20k}{41}$$

$$V_{gs2} = V_{g2} - V_{s2} = (0V + -g_{m1} V_{gs1} \cdot 20k\Omega) - (g_{m2} V_{gs2} \cdot 40k\Omega \parallel 1k\Omega)$$

$$\therefore V_{gs2} = -g_{m1} V_{gs1} \cdot 20k\Omega - g_{m2} V_{gs2} \cdot 0.98k\Omega$$

$$\therefore V_{gs2} (1 + g_{m2} \cdot 0.98k\Omega) = -g_{m1} V_{gs1} \cdot 20k\Omega$$

$$\therefore V_{gs2} = \frac{-g_{m1} V_{gs1} \cdot 20k\Omega}{1 + g_{m2} \cdot 0.98k}$$

$$V_{gs1} = V_{g1} - V_{s1} = 0V - (V_{sig} + g_{m1} V_{gs1} \cdot 30k)$$

$$\therefore V_{gs1} = -V_{sig} - g_{m1} V_{gs1} \cdot 30k$$

$$\therefore V_{gs1} (1 + g_{m1} \cdot 30k) = -V_{sig}$$

$$V_{gs1} = \frac{-V_{sig}}{1 + g_{m1} \cdot 30k}$$

$$\text{All together: } V_o = \frac{20k \cdot g_{m2}}{41} \left( \frac{+g_{m1} \cdot 20k}{1 + g_{m2} \cdot 0.98k} \left( \frac{+V_{sig}}{1 + g_{m1} \cdot 30k} \right) \right)$$

$$V_o = \frac{g_{m1} g_{m2} \cdot 20k \cdot 20k}{41 (1 + g_{m2} \cdot 0.98k) (1 + g_{m1} \cdot 30k)} V_{sig}$$

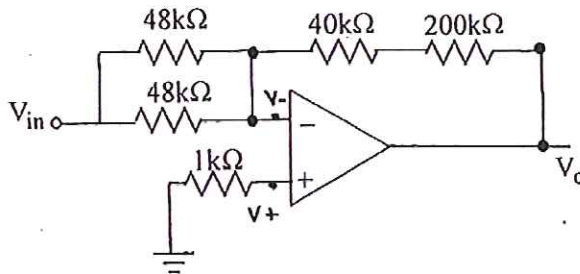
$$\frac{V_o}{V_{sig}} = \frac{(1m)(25m)20k \cdot 20k}{41 (1 + (25m)(0.98k)) (1 + (1m)(30k))} = \frac{100000}{32410.5} = \boxed{3.09 \frac{V}{V}}$$

### Problem 4 – (20 points)

You are given the following characteristics for a real amplifier:

Input offset voltage,	$V_{ios} = 2\text{mV}$
Input Resistance,	$R_i = 2\text{M}\Omega$
Unity-gain bandwidth,	$f_T = 40\text{MHz}$
Output swing limits,	within 2Volts of power supply
Slew Rate,	$SR = 5 \frac{\text{V}}{\mu\text{sec}}$

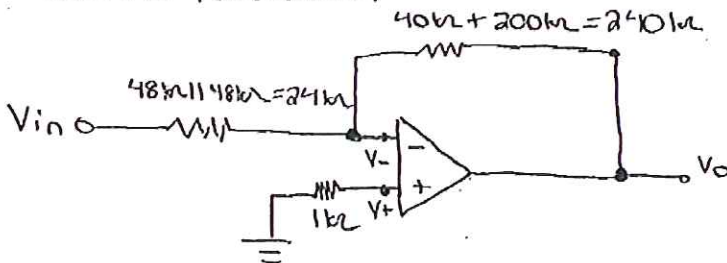
The following circuit is powered at  $\pm 9\text{V}$ :



- (3 points) What value is the ideal gain?
- (10 points) What is the bandwidth of the circuit considering both the Unity-gain bandwidth limitations and the slew rate effect for an input of  $V_{in} = 0.001 \sin(\omega t)$ ?
- (4 points) For  $V_{in} = 0.001 \sin(\omega t)$ , what is the maximum and minimum values seen at the output considering only the input offset voltage?
- (3 points) How should the circuit above be modified (do not remove any resistors) to minimize the effect of the input bias current? Draw the schematic of the modified circuit and state values of added component(s).

a) Ideal op-amp:  $V_+ = V_-$  (see schematic), no current into op-amp.

Combine resistances:



This is just an inverting amplifier, so the gain is

$$\frac{V_o}{V_{in}} = -\frac{240\text{k}\Omega}{24\text{k}\Omega} = -10 \frac{\text{V}}{\text{V}}$$

or  $(10 \frac{\text{V}}{\text{V}}, \text{inverting})$ .

b) Effect due to unity-gain bandwidth:

$$f_c \cdot \text{gain} = f_T \rightarrow f_c = \frac{f_T}{\text{gain}} = \frac{40\text{MHz}}{10} = 4\text{MHz}$$

Effect due to slew rate:  $f_{\text{max}} = \frac{SR}{V_{pp} \cdot \pi}$ , where  $SR = \text{slew rate}$ ,  
 $V_{pp} = 2 \cdot \text{amplitude of output signal}$

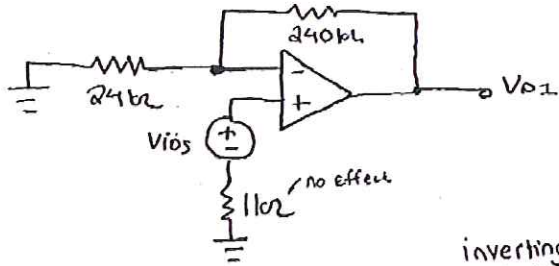


Amplitude of output signal:  $\text{gain} \cdot 0.001 \text{ V} = 10 \cdot 1 \text{ mV} = 10 \text{ mV}$

So,  $V_{pp} = 2(10 \text{ mV}) = 20 \text{ mV}$  amplitude of input signal

$f_{\text{max}} = \frac{5 \text{ V}}{1 \mu\text{s} \cdot 20 \text{ mV} \cdot \pi} \approx 79.6 \text{ MHz}$ . Smaller frequency determines the bandwidth, so bandwidth is 4 MHz.

c) Use superposition.  $V_{ios}$  on,  $V_{in}$  off.



This is just a noninverting amplifier.

$$\frac{V_{o1}}{V_{ios}} = \left(1 + \frac{240 \text{ k}\Omega}{24 \text{ k}\Omega}\right) = (1+10) = 11$$

$$\therefore V_{o1} = 11 \cdot V_{ios} = 11 \cdot 2 \text{ mV} = 22 \text{ mV}$$

$V_{in}$  on,  $V_{ios}$  off:  $V_{o2} = \text{gain} \cdot V_{in} = -10 \cdot (0.001 \sin(\omega t)) = -10 \text{ mV} \cdot \sin(\omega t)$

Combine results:  $V_o = V_{o1} + V_{o2} = 22 \text{ mV} - 10 \text{ mV} \sin(\omega t)$

Max value:  $22 \text{ mV} + 10 \text{ mV} = 32 \text{ mV}$   
 Min value:  $22 \text{ mV} - 10 \text{ mV} = 12 \text{ mV}$

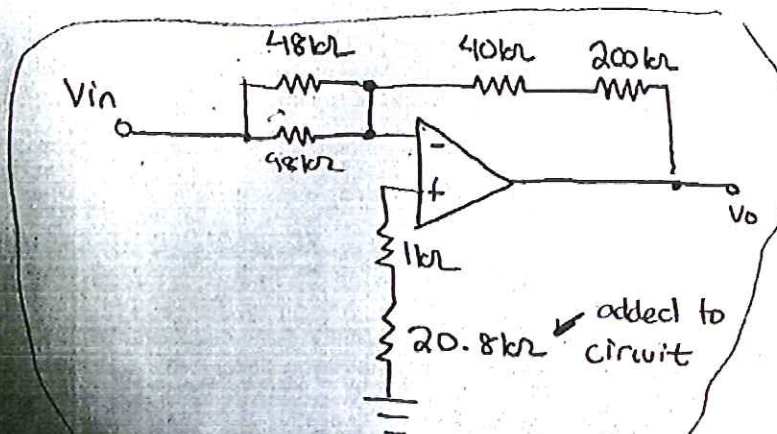
d) Match the impedance seen by each terminal of the op-amp.

- terminal sees impedance  $R_{TH} = 24 \text{ k}\Omega \parallel 240 \text{ k}\Omega = 24 \text{ k}\Omega \cdot \frac{1}{11} = 24 \text{ k}\Omega \cdot \frac{10}{11} \approx 21.8 \text{ k}\Omega$

+ terminal sees impedance  $1 \text{ k}\Omega$ . If we add a  $20.8 \text{ k}\Omega$  resistor in series with this  $1 \text{ k}\Omega$  resistor, then the impedances will match.

ECE 2280 Midterm #1

Circuit:



Name \_\_\_\_\_

Scores:  
 Prob 1 \_\_\_\_\_ of a possible 25pts

Prob 2 \_\_\_\_\_ of a possible 25pts

Prob 3 \_\_\_\_\_ of a possible 30pts

Prob 4 \_\_\_\_\_ of a possible 20pts

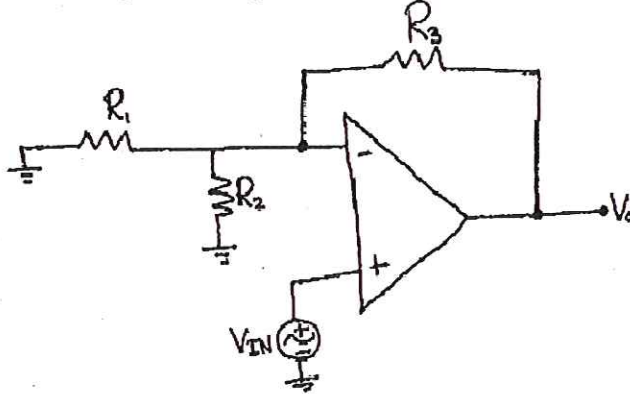
Total \_\_\_\_\_ of a possible 100 pts

**Problem 4 – (20 points)**

You are given the following characteristics for a real amplifier:

- Input offset voltage,  $V_{ios}=4mV$
- Input Resistance,  $R_i=2M\Omega$
- Unity-gain bandwidth,  $f_T=10MHz$
- Output swing limits, within 2Volts of power supply
- Slew Rate,  $SR=5 \frac{V}{\mu sec}$

The following circuit is powered at  $\pm 12V$ :



- a) State the equation for  $V_o$ . Include no more than  $V_{IN}$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .
- b) If  $R_1=R_2=10k$  and  $R_3=100k$ , what is the bandwidth of the circuit. Consider both the effect due to slew rate (use the maximum output value possible) compared to the effect due to the unity gain bandwidth.
- c) For  $V_{in}=0.001\sin(2\pi 90kt)$ , what is the PEAK(not peak to peak) value at the output considering the input offset voltage?
- d) How should the circuit above be modified to minimize the effect of the input bias current? Draw the schematic of the modified circuit and state values of added component(s).

a) 
$$V_o = \left[ \frac{R_3}{R_2 \parallel R_1} + 1 \right] \cdot V_{IN}$$

b) 
$$V_o = \left[ \frac{100k}{5k} + 1 \right] \cdot V_{IN}$$

$$\frac{V_o}{V_{IN}} = 21 \Rightarrow f_{3dB} = \frac{10M}{21} = 476kHz$$

$$f_{max} = \frac{5}{1 \times 10^{-6}} \cdot \frac{1}{(20 \cdot \pi)} \approx 80kHz$$

c) 
$$V_{out\ peak} = 21m + 4m(21) = 105mV_{peak}$$

