Effect of Poles not at the origin on Phase Angle:

Poles not at the origin, like \( \frac{1}{1 + \frac{j \omega}{p_1}} \), have no phase shift for frequencies much lower than \( p_1 \), have a -45 deg shift at \( p_1 \), and have a -90 deg shift for frequencies much higher than \( p_1 \).

\[
\angle \text{TF} \quad 0.1p_1 \quad 1p_1 \quad 10p_1 \quad 100p_1 \quad \omega
\]

-45 deg

-90 deg

To draw the lines for this type of term, the transition from 0\(^\circ\) to -90\(^\circ\) is drawn over 2 decades, starting at 0.1\( p_1 \) and ending at 10\( p_1 \).

When drawing the phase angle shift for not-at-the-origin zeros and poles, first locate the break frequency of the zero or pole. Then start the transition 1 decade before, following a slope of \( \pm 45^\circ \) /decade. Continue the transition until reaching the frequency one decade past the break frequency.

**SUMMARY OF STRAIGHT-LINE APPROXIMATION PROCEDURE STEPS (NO COMPLEX):**

(Note that a decade is a **multiple** of 10 - 1, 10, 100, 1000, etc)

1. Rearrange the equation into standard form:
   \[
   H(s) = \frac{Kz_1z_2\cdots(\frac{s}{z_1} + 1)(\frac{s}{z_2} + 1)\cdots}{p_1p_2\cdots(\frac{s}{p_1} + 1)(\frac{s}{p_2} + 1)\cdots}
   \]

   where \( K, z_1, z_2, \) etc are all constant values.

2. Determine the poles and zeros.

   **Note:** If there are more than one poles/zeros at the same break frequency (say there are \( r \)), just multiply the slope/phase changes by \( r \). (ex. \((1 + s/10)^2 \Rightarrow \) it is a negative zero (numerator) and so it will change the slope by 2*20dB/dec and have a 2*45\(^\circ\) slope/dec.

3. Draw the magnitude plot:
   a. Determine starting value:
      
      **Case 1:** No pole or zero at the origin:
      \[
      \text{starting value} = 20 \log_{10}(\frac{Kz_1z_2\cdots}{p_1p_2\cdots})
      \]
      
      **Case 2:** A pole or zero at the origin:
      
      * Pick a frequency value less than the lowest pole or zero value.

Dr. Rasmussen
Plug in the frequency in the standard form equation above and take the magnitude. *This value is for that frequency only.* There is a constant slope going through this point. +20dB/dec slope if the location is a zero. -20dB/dec slope if the location is a pole.

b. Begin at the starting point. Start with the slope (0 slope if a constant, +20dB/dec slope if zero at origin, -20dB/dec slope if pole at origin). From left to right, at each zero add +20dB/dec to the current slope and at each pole -20dB/dec. Continue through each frequency.

4. Draw the phase plot:
   a. Determine the starting value:
      
      **Case 1: No pole or zero at the origin:**
      
      If constant>0 then starting value = 0°
      
      If constant<0 then starting value = ±180°

      **Case 2: A pole or zero at the origin:**
      
      starting value = +90° if zero at origin
      
      starting value = -90° if pole at origin

   b. Label each range of frequency according to the following(suggest putting on graph):
      
      zero => from 1 decade before frequency to 1 decade after frequency: +45°slope/dec
      
      pole => from 1 decade before frequency to 1 decade after frequency: -45°slope/dec
      
      (eg if ω = 10 and is a pole then range is 1<ω<100 with a slope of -45°slope/dec)

   c. Look at each frequency range that has a slope. Add all slopes within that region. From left to right: start with starting value and slope of 0, continue until first region of change. Add all slopes within that region. Continue until the end is met. If no slope during a region the slope is constant (0).
BODE PLOTS IN MATLAB

Examples using three different methods applied to the transfer function from Prelab 1:

\[ TF = \frac{20000}{s + 20000} \]

**Method 1: Easiest (If you have the Control Toolbox in Matlab)**

```matlab
s=tf('s');
H = (20000/(s+20000));
Bode(H)
grid on
```

**Method 2: Annalisa's Way (With no Control Toolbox...)**

```matlab
% Expand the numerator and denominator of your transfer function by multiplying out the terms. Then
% make an array of the coefficients of the numerator and denominator of the transfer function in descending
% order of powers. Example: if numerator is As^2+Bs+C, array will be num=[A B C]. Note that the arrays
% for the numerator and denominator must be equal in length.
numTF=[0 20000];
denomTF=[1 20000];
w=0:10:10e4;

% Function 'freqs' gives the frequency response in the s-domain
Y=freqs(numTF,denomTF,w);
y1=abs(Y);
y2=angle(Y);

subplot(2,1,1)
semilogx(w,20*log10(y1))
grid on
ylabel('Magnitude (dB)')
title('Bode Diagram')

subplot(2,1,2)
semilogx(w,y2*(180/pi))
grid on
ylabel('Phase (deg)')
xlabel('Frequency (Rad/s)')
```
Method 3: Dr. Rasmussen's Way (With no Control Toolbox....)

Function 'logspace' creates an array of 200 points from -1 to 10^5 spaced logarithmically

```matlab
w=logspace(-1,5,200);
MagH=sqrt(w.^2+20000^2)/sqrt(w.^2+20000^2);

MagHdb=20*log10(MagH);
PhaseHRad=atan(w/20000);
PhaseHDeg=PhaseHRad*180/pi;

subplot(2,1,1)
semilogx(w,MagHdb)
ylabel('20 log10(|TF|) [dB]')
title('Bode Diagram')
grid on

subplot(2,1,2)
semilogx(w,PhaseHDeg)
xlabel('frequency [rad/s]')
ylabel('Phase Angle [deg]')
grid on
```
Example

\[ H(s) = \frac{100(s+100)(s+10)}{s^2(s+10k)} = \frac{100 \left[ (s+100) \left( \frac{s}{100} + 1 \right) \right] (s+10)}{s^2(s+10k)(s/10k + 1)} \]

zeros: 100, 10

poles: at origin, 10k

slope is -40dB/dec

\[ w=1 \text{ rad/sec} \]

\[ w=10 \text{ dB} \]

\[ w=100 \text{ dB} \]

\[ w=10k \text{ dB} \]

\[ \text{Phase: } (\pi) \begin{align*}
    & 1 < w < 10 : +45^\circ \text{ slope/dec} \\
    & 10 < w < 1k : -45^\circ \text{ slope/dec} \\
    & 1k < w < 100k : -45^\circ \text{ slope/dec} \\
    \end{align*} \]

starting phase: \( 2\times (-90^\circ) = -180^\circ \)
Example

From circuit before: \( \frac{V_o}{V_i} = \frac{133n \cdot S}{(266.7nS + 1)} \)

1 zero at origin

pole: \( \frac{1}{266.7n} = 3.75 \text{Meg} \)

\( \omega = 1: 20 \log \left[ \frac{133n(1)}{\sqrt{266.7n^2 + 1^2}} \right] = -138 \text{dB} \)

\( \omega = 3.75 \text{Meg} \Rightarrow 20 \log \left[ \frac{133n(3.75 \text{Meg})}{\sqrt{(266.7n \times 3.75 \text{Meg})^2 + 1^2}} \right] \)

\[ = 20 \log (0.358) = -9 \text{dB} \]

phase: +90°

375k < \( \omega \) < 37.5 Meg: -45° slope/dcr
Example 10: Bode Plots:

\[
\frac{V_o}{V_i} = \frac{4 \times 10^6 \cdot s}{30 \left( \frac{2}{20} \cdot \frac{m}{s} + 1 \right)} = \frac{133n \cdot s}{2(\omega_0 \cdot 7n \cdot s + 1)}
\]

Break frequency \( \Rightarrow (\omega_0 + 1) \Rightarrow \omega_0 = \frac{1}{\omega_0} \text{ rad/sec} = 3.75 \text{ Mega} \)

\[ w = 3.75 \text{ Mega} \Rightarrow 20 \log \left( \frac{133n \times 3.75 \text{ Mega}}{\sqrt{(266.7 n \times 3.75 \text{ Mega})^2 + 1}} \right) \]

\[ \approx 20 \log (1.353) = -90 \text{ db} \]

Because of zero at origin, start phase at 0°

Note that this circuit only operates at frequencies above 3.75 MHz rad/sec = 597 KHz

Example 11:

Analyze the following circuit to find the transfer function \( \frac{V_i}{V_o} \). Solve the circuit symbolically first and then with their values. Sketch transfer function using a straight-line approximation procedure.

\[ \frac{V_i}{V_o} = \frac{R_1 (R_1 + C_1 s)}{R_2 + (R_1 R_2 + C_2 s)} + \frac{R_2 R_1}{C_2 s} \]

\[ \approx \frac{R_2 R_1}{R_2 + R_1 + C_2 s} \]

\[ \frac{V_i}{V_o} = \frac{R_2 R_1}{R_2 + R_1 + C_2 s} \]

This circuit only operates above 20 MHz and below 32 MHz.