1. Calculate Bode Plots of the following:

(a) \( H(s) = \frac{(s + 100)}{(s + 10^3)(s + 10^4)} \)

- Start value: \( H(0) = \frac{100}{(10^3*10^4)} = 10^{-5} \Rightarrow 20\log_{10}(10^{-5}) = -100\text{dB} \)
- Critical frequencies:
  - \( \omega=100 \) – (negative zero) \( \Rightarrow +20\text{dB/dec}, +45^\circ \text{ slope/dec} \) (over 2 decades \( \omega=10->1,000 \))
  - \( \omega=1,000 \) – (negative pole) \( \Rightarrow -20\text{dB/dec}, -45^\circ \text{ slope/dec} \) (over 2 decades \( \omega=100->10,000 \))
  - \( \omega=10,000 \) – (negative pole) \( \Rightarrow -20\text{dB/dec}, -45^\circ \text{ slope/dec} \) (over 2 decades \( \omega=1,000->100,000 \))
2. Calculate the Bode plot for the following:

\[ H(s) = \frac{10}{s^2(s+100)} \]

(a) \( n=2 \) (the number of poles or zeros at the origin – 2 poles at the origin)

- **gain:** \( K = H(s) \cdot s^2 \mid_{s=0} = \frac{10}{100} = 0.1 \Rightarrow 20 \log_{10}(0.1 \cdot 1^{-2}) = -20 \text{dB} \)
  - choose \( \omega_{\text{start}} = 0.1 \) and you get \( 20 \log_{10}(0.1 \cdot 1^{-2}) = 20 \text{dB} \)
- **phase:** \( K>0, \ n \cdot 90^\circ = -2 \cdot 90^\circ = -180^\circ \)

(b) critical frequencies:

- \( \omega = 0 \) – (pole at origin) \( \Rightarrow -40 \text{dB/dec}, -180^\circ \) start
- \( \omega = 1,00 \) – (negative pole) \( \Rightarrow -20 \text{dB/dec}, -45^\circ \) slope/dec (over 2 decades \( \omega = 10 \rightarrow 1,000 \))
Problem 2 – (25 points)  

a) Sketch the Bode (both magnitude & phase) plot for: (label your axis and show all your work)

\[ H(s) = \frac{(100)(s+100)(s+10)}{(s^2)(s+10,000)} \]

b) What is the estimated magnitude value at \( \omega = 1 \) rad/sec:

\[ K = \lim_{\omega \to 0} H(\omega) = \frac{100}{10,000} = 1 \]

\( \omega_{\text{start}} = 1 \)

\( n = -2 \)

\[ 20 \log (10(1)^2) = 20 \text{dB} \]

c) For the magnitude plot, what is the slope of the line going through \( \omega = 1 \) rad/sec:

\[ n \ast 20 \text{ dB/dec} = -40 \text{ dB/dec}. \]

d) What is the estimated phase value at \( \omega = 1 \) rad/sec:

\[ K > 0 \quad \therefore \quad \text{phase} = -180 \text{ degrees} \]

e) For the phase plot, what is the slope of the line to the left of \( \omega = 1 \) rad/sec:

\[ \bigcirc \]

f) For the phase plot, what is the slope of the line to the right of \( \omega = 1 \) rad/sec:

\[ +45^\circ \text{ slope/dec}. \]

g) List the three frequencies other than 0 where the bode plots will have a change in slope (or value):

<table>
<thead>
<tr>
<th>( \omega )</th>
<th>Change in Slope</th>
<th>Change in Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 100,000 )</td>
<td>neg. pole ( \to \omega = 1,000 \to 100,000 )</td>
<td>( -90^\circ \to -45^\circ \text{ dB/dec}. )</td>
</tr>
<tr>
<td>( 100 )</td>
<td>+20 dB/dec. ( \to \omega = 10 \to 1,000 )</td>
<td>+45°</td>
</tr>
<tr>
<td>( 10 )</td>
<td>+20 dB/dec. ( \to \omega = 1 \to 100 )</td>
<td>+45°</td>
</tr>
</tbody>
</table>
\[ H(s) = \frac{(100)(s + 100)(s + 10)}{(s^2)(s + 10,000)} \]
Sketch the Bode (both magnitude & phase) plot for: \( H(s) = \frac{(1\times10^8)(s+10)}{(s+(1\times10^4))(s+(1\times10^3))} \)

\[ H(0) = \frac{(1\times10^8)(10)}{(1\times10^4)(1\times10^5)} = 1 \]

\[ 20 \log_{10}(1) = 0 \text{dB} \]

Phase \(\Rightarrow 0^\circ\)

Critical frequencies:
- \(\omega = 10\): +20 dB/dec, +45° slope/dec (over 2 decades)
- \(\omega = 1\times10^4\): -20 dB/dec, -45° slope/dec (over 2 decades)
- \(\omega = 1\times10^5\): -20 dB/dec, -45° slope/dec (over 2 decades)