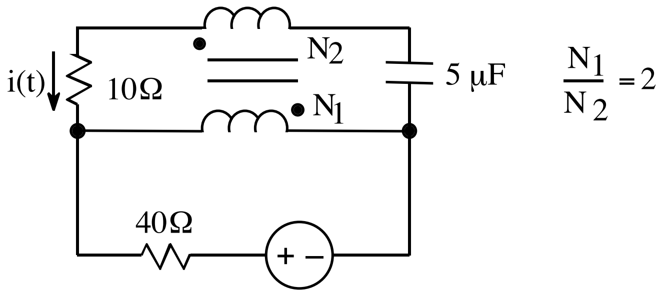
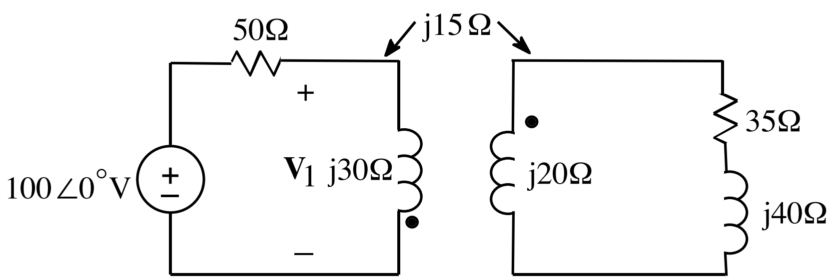
3. (40 points)



a. Write a numerical time-domain expression for the current i(t).



b. Calculate $\mathbf{V}_1$.

ans: a)

$$i(t) = \frac{3}{4}\sqrt{2}\cos(2\mathrm{k}t - 135^\circ)\mathrm{A}$$

b)

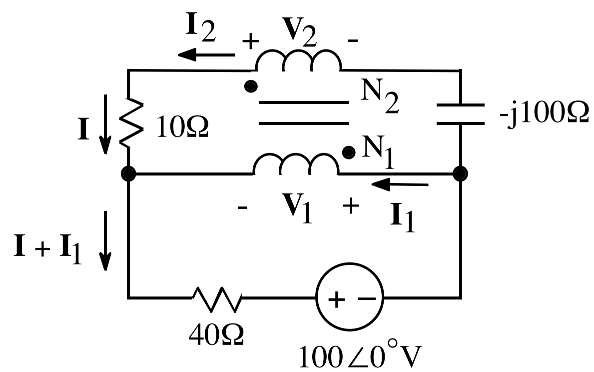
$$\mathbf{V}_1 = 46.7\angle 58.8^\circ \mathrm{V}$$

sol'n: (a) We assume an ideal transformer since we are only given the turns ratio N_1/N_2. We want to draw the circuit in the s-domain with labels for $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$, and $\mathbf{V}_2$.

$\omega = 2k$ from $V_s = 100\cos(2k\cdot t)\text{V}$

$$\frac{1}{j\omega C} = \frac{1}{j2k\cdot 5\mu}\Omega = -\frac{j}{10m}\Omega = -j100\Omega$$

Our circuit diagram in the s-domain:



Note: We measure $\mathbf{V}_1$ and $\mathbf{V}_2$ with plus signs at dots on transformer.

$\mathbf{I}_1$ (primary side) flows <u>into</u> dotted terminal.

$\mathbf{I}_2$ (secondary side) flows <u>out</u> of dotted terminal.

For the above definitions of $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$, and $\mathbf{V}_2$, we have ideal transformer equations without minus signs:

$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2} \qquad \mathbf{I}_1 N_1 = \mathbf{I}_2 N_2$$

Now we write equations for mesh (current) loops. (We could also use the node-voltage method.)

We observe that $\mathbf{I} = \mathbf{I}_2$ in the top loop, and since $\mathbf{I}_2$ is flowing on the outer edge of the circuit, (where there is no circuit on the other side to cause a summation of mesh currents through components), we see that $\mathbf{I}$ is also the mesh current for the top loop.

The current mesh equation for the top loop is (the sum of V drops around the loop):

(1) $\mathbf{V}_2 - \mathbf{I}\cdot 10\Omega + \mathbf{V}_1 - \mathbf{I}\,(-j100\Omega) = 0\text{V}$

Note: $\mathbf{I}$ must also flow up through C. What $\mathbf{I}$ goes down, must come up. (Otherwise, we would accumulate charge in the bottom half of the circuit.)

The mesh current for the bottom loop will be $\mathbf{I} + \mathbf{I}_1$. This current is flowing on the outside edge of the circuit in the bottom loop.

Our mesh loop equation for the bottom (i.e. sum of V drops around loop) is:

$$-\mathbf{V}_1 - (\mathbf{I} + \mathbf{I}_1)40\Omega - 100\angle 0^\circ = 0\,\mathrm{V}$$

or

$$(2) \quad -\mathbf{V}_1 + (\mathbf{I} + \mathbf{I}_1)40\Omega + 100\angle 0^\circ = 0\mathrm{V}$$

Now we use the ideal transformer equations to eliminate all but two unknowns:

$$\mathbf{V}_1 = \frac{\mathrm{N}_1}{\mathrm{N}_2}\mathbf{V}_2 = 2\mathbf{V}_2, \quad \mathbf{I}_1 = \frac{\mathrm{N}_2}{\mathrm{N}_1}\mathbf{I}_2 = \frac{\mathrm{N}_2}{\mathrm{N}_1}\mathbf{I} = \frac{\mathbf{I}}{2}$$

Substituting these into Eq. 1 and Eq. 2 gives two equations in two unknowns:

$$(1') \quad \mathbf{V}_2 - \mathbf{I}\cdot 10\Omega + 2\mathbf{V}_2 - \mathbf{I}(-j100\Omega) = 0V$$

$$(2') \quad \mathbf{2V}_2 + \left(I + \frac{I}{2}\right)40\Omega + 100\angle 0^\circ = 0\mathrm{V}$$

Now we solve for **I**. From Eq. 2':

$$\mathbf{V}_2 = -\left(\frac{\frac{3}{2}\mathbf{I}\cdot 40\Omega + 100}{2}\right) = -30\cdot\mathbf{I} - 50\mathrm{V}$$

Eq. 1' rearranged is

$$3\mathbf{V}_2 + (-10 + j100)\mathbf{I} = 0\mathrm{V}.$$

By substituting for $\mathbf{V}_2$, and doing the algebra, we find **I**:

$$3(-30\cdot\mathbf{I} - 50) + (-10 + j100)\mathbf{I} = 0\mathrm{V}$$

$$(-100 + j100)\mathbf{I} - 150 = 0\mathrm{V}$$

$$\mathbf{I} = \frac{150}{-100 + j100}\mathrm{A} = \frac{3}{-2 + j2}\mathrm{A} = -\frac{3}{2}\,\frac{1}{1-j}\mathrm{A}$$

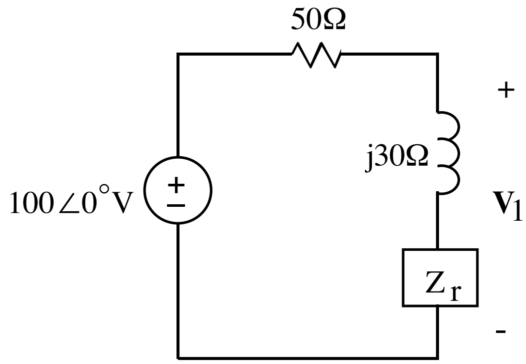
$$\mathbf{I} = -\frac{3}{4}\sqrt{2}\angle 45^\circ\ \mathrm{A}$$

$$\mathbf{I} = \frac{3}{4}\sqrt{2}\angle -180^\circ + 45^\circ\ \mathrm{A}$$

$$\mathbf{I} = \frac{3}{4}\sqrt{2}\angle -135^\circ\ \mathrm{A}$$

$$\therefore \quad i(t) = \frac{3}{4}\sqrt{2}\cos(2\mathrm{k}t - 135^\circ)\mathrm{A}$$

sol'n: (b) We use the idea of reflected impedance for a linear transformer:



The formula for reflected impedance with a linear transformer is

$$Z_r = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$$

Here, we have $R_2 = 0\Omega$, $j\omega L_2 = j20\Omega$, and $Z_L = 35\Omega + j40\Omega$.

$j\omega M = j15\Omega$ is the mutual inductance. $\therefore\ \omega M = 15\Omega$

$$\therefore \quad Z_r = \frac{225\Omega^2}{j20 + 35 + j40\Omega} = \frac{225}{35 + j60}\Omega = \frac{45}{7 + j12}\Omega$$

$$Z_r = \frac{45}{193}(7 - j12)\Omega$$

Using the equivalent model for primary side, as shown above, we have

$$\mathbf{V}_1 = \frac{j30\Omega + Z_r}{50 + j30 + Z_r} \cdot 100\angle 0^\circ \mathrm{V}$$

$$\mathbf{V}_1 = \frac{j30 + \dfrac{45}{193}(7 - J12)}{50 + j30 + \dfrac{45}{193}(7 - J12)} \cdot 100\angle 0^\circ \mathrm{V}$$

We can factor out a 5 from top and bottom:

$$\mathbf{V}_1 = \frac{j(193)30 + 45(7 - j12)}{(193)\cdot \cancelto{10}{50} + j(193)\cancelto{6}{30} + \cancelto{9}{45}(7 - j12)} \cdot \cancelto{20}{100} = \frac{315 + j5250}{1993 + j1050} \cdot 20\mathrm{V}$$

Evaluating the expression and converting to polar form gives

$$\mathbf{V}_1 = 46.7\angle 58.8^\circ \mathrm{V}.$$