ECE

2270

UNIT 2

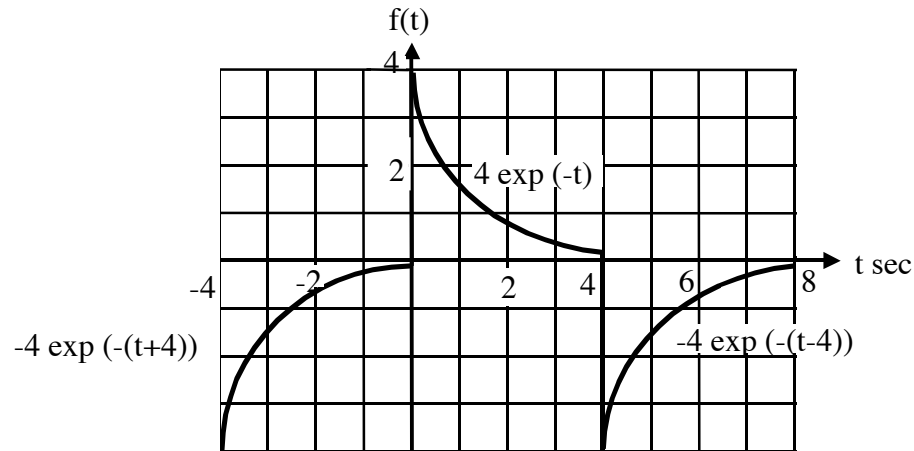PRACTICE EXAM SOLUTION Prob 2

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2. (40 points)

Find the coefficients for the Fourier series of the above function. The period of the function is 8 sec. From integral tables or a calculator we have:

$$\int e^{ax} \cos(bx)dx = \frac{e^{ax}[a\cos(bx) + b\sin(bx)]}{a^2 + b^2}$$

$$\int e^{ax} \sin(bx)dx = \frac{e^{ax}[a\sin(bx) - b\cos(bx)]}{a^2 + b^2}$$

Ans:

$$a_k = \begin{cases} \dfrac{2\left(1+e^{-4}\right)}{1+\left(\dfrac{k\pi}{4}\right)^2} & k \text{ odd} \\ \\ 0 & k \text{ even} \end{cases}$$

$$b_k = \begin{cases} \dfrac{\dfrac{k\pi}{2}\left(1+e^{-4}\right)}{1+\left(\dfrac{k\pi}{4}\right)^2} & k \text{ odd} \\ \\ 0 & k \text{ even} \end{cases}$$

Sol'n: Since $f(t)$ has equal area above and below the horizontal axis, we conclude that the DC offset is zero: $a_v = 0$.

Since $f(t)$ is neither even nor odd, we will have nonzero a_k and b_k terms. $f(t)$ does have shift-flip symmetry, however, so even-numbered terms for a_k and b_k will be zero. Furthermore, we need only compute integrals over the first half of the function and double them to find values of a_k and b_k for k odd.

We have the following formulas based on shift-flip symmetry:

$$a_k = 2\frac{2}{T}\int_0^{T/2} 4e^{-t}\cos(k2\pi t/T)dt$$

$$b_k = 2\frac{2}{T}\int_0^{T/2} 4e^{-t}\sin(k2\pi t/T)dt$$

Using the integrals given in the problem and $T = 8$, we have (for k odd)

$$a_k = 2\int_0^4 e^{-t}\cos(k\pi t/4)dx = 2\cdot \frac{e^{-t}\left[-\cos(k\pi t/4) + \dfrac{k\pi}{4}\sin(k\pi t/4)\right]\Big|_0^4}{(-1)^2 + \left(\dfrac{k\pi}{4}\right)^2}$$

$$b_k = 2\int_0^4 e^{-t}\sin(k\pi t/4)dx = 2\cdot \frac{e^{-t}\left[-\sin(k\pi t/4) - \dfrac{k\pi}{4}\cos(k\pi t/4)\right]\Big|_0^4}{(-1)^2 + \left(\dfrac{k\pi}{4}\right)^2}$$

Using $\sin(0) = 0$, $\sin(k\pi) = 0$, and $\cos(0) = 1$, and substituting $t = 4$ for the upper limit, gives

$$a_k = 2\cdot \frac{e^{-4}\left[-\cos(k\pi)\right] + 1}{(-1)^2 + \left(\dfrac{k\pi}{4}\right)^2}$$

$$b_k = 2\cdot \frac{\left(-\dfrac{k\pi}{4}\right)\left[e^{-4}\cos(k\pi) - 1\right]}{(-1)^2 + \left(\dfrac{k\pi}{4}\right)^2}$$

We simplify further by noting that $\cos(k\pi) = -1$ when k is odd. With this substitution, we get the final answer.