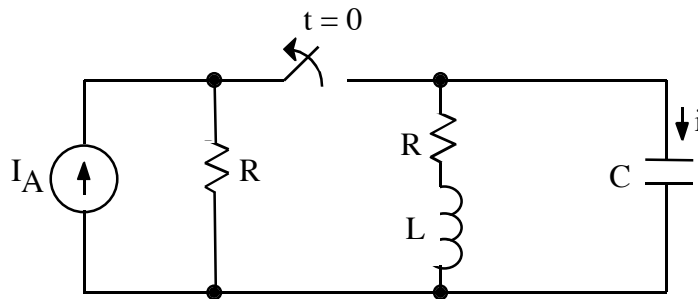


3. (50 points)



$$\begin{aligned}
 I_A &= 1 \text{ A} \\
 R &= 2400 \, \Omega \\
 L &= 200 \, \mu\text{H} \\
 C &= 50 \text{ pF}
 \end{aligned}$$

- After being closed for a long time, the switch is opened at $t = 0$. Write a numerical time-domain expression for $i(t)$, the current through the capacitance. This expression must not contain any complex numbers.
- State whether $i(t)$ is underdamped, overdamped, or critically damped.

ans: a) $i(t > 0) = \left(-\frac{1}{2} \cos 8Mt - \frac{3}{8} \sin 8Mt \right) e^{-6Mt} \text{ A}$
 b) Underdamped.

sol'n: (a) When the switch is open, we have series RLC.

$$\alpha = \frac{R}{2L} = \frac{2.4\text{k}}{2 \cdot 200\mu} = \frac{1.4\text{k}}{400} \text{ M/s} = 6 \text{ M/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{200\mu \cdot 50\text{p}} = \frac{1}{10\text{k} \mu\text{p}} = \frac{1\text{M} \cdot \text{M} \cdot \text{M}}{10\text{k}} = 100 \text{ M}^2/\text{s}^2$$

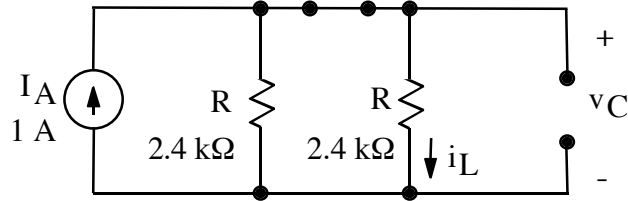
$$\therefore \omega_o = 10 \text{ M/s}$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{(10 \text{ M})^2 - (6 \text{ M})^2} = 8 \text{ M/s} \quad (6^2 + 8^2 = 10^2)$$

Now find initial condition (i.e. i and di/dt , or v and dv/dt at $t = 0^+$).

$$\left. \begin{aligned}
 i_L(t = 0^+) &= i_L(t = 0^-) \\
 v_C(t = 0^+) &= v_C(t = 0^-)
 \end{aligned} \right\} \text{cannot change instantly}$$

Circuit for $t = 0^-$: L's = wires, C's = open circuits.



$$i_L(t=0^-) = \frac{1}{2} \text{ A} \quad (\text{current divider})$$

$$v_C(t=0^-) = I_A R \parallel R = I_A \cdot \frac{R}{2} = 1\text{A} \cdot 1.2 \text{ k}\Omega = 1.2 \text{ kV}$$

$$\therefore i_L(t=0^+) = \frac{1}{2} \text{ A}, \quad v_C(t=0^+) = 1.2 \text{ kV}$$

After the switch is open, $i = -i_L$ since C and L are in series.

\therefore Solve for i_L and then change the sign. Note that i_L is the variable in the differential equation for a series RLC. Thus, we know how to find it.

We also need

$$\left. \frac{di_L(t)}{dt} \right|_{t=0^+}.$$

Use V-loop for RLC at $t = 0^+$:

$$L \frac{di_L}{dt} + i_L R - v_C = 0 \text{ V}.$$

Note that at $t = 0^+$, ($i_R = i_L$ since R, L in series),

$$\left. \frac{di_L}{dt} \right|_{t=0^+} = \frac{-i_L(t=0^+)R + v_C(t=0^+)}{L}$$

$$\left. \frac{di_L}{dt} \right|_{t=0^+} = \frac{-\frac{1}{2} \text{ A} \cdot 2.4 \text{ k}\Omega + 1.2 \text{ kV}}{200 \text{ }\mu\text{H}}$$

$$\left. \frac{di_L}{dt} \right|_{t=0^+} = 0 \text{ A/s}$$

Now use general underdamped solution:

$$i_L(t) = (B_1 \cos \omega_d t + B_2 \sin \omega_d t) e^{-\alpha t}$$

$$B_1 = i_L(t = 0^+), \quad -\alpha B_1 + \omega_d B_2 = \left. \frac{di_L}{dt} \right|_{t=0^+} = 0 \text{ A/s}$$

$$\therefore B_1 = \frac{1}{2} \text{ A}, \quad B_2 = \frac{\alpha B_1}{\omega_d} = \frac{6M}{8M} \frac{1}{2} \text{ A} = \frac{3}{8} \text{ A}$$

$$\therefore i(t > 0) = \left(\frac{1}{2} \cos 8Mt + \frac{3}{8} \sin 8Mt \right) e^{-6Mt} \text{ A}$$

sol'n: (b) $\omega_o > \alpha \Rightarrow$ underdamped