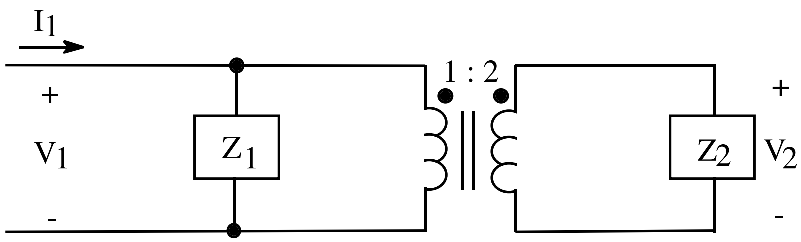
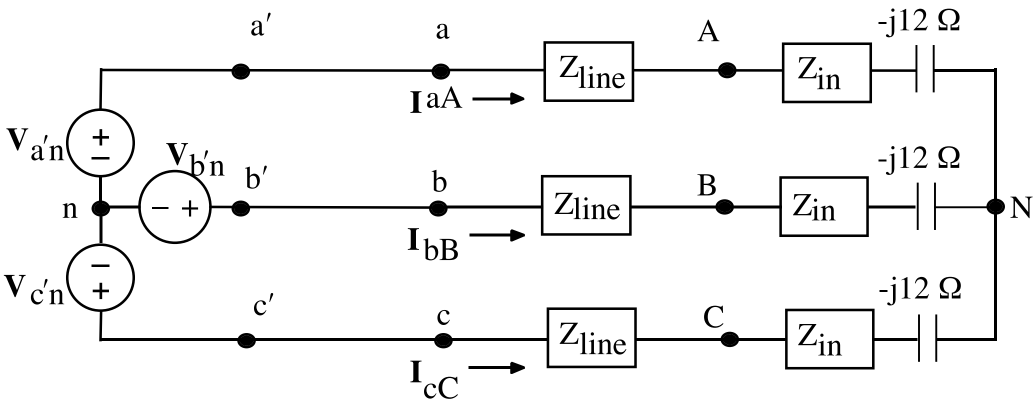
4. (50 points)




$Z_1 = (5 - j5)\Omega$ $Z_2 = (20 + j20)\Omega$

a. Find the input impedance, $z_{in} = \mathbf{V}_1/\mathbf{I}_1$, for the above circuit.

b. Using $z_{in}$ from (a), find a numerical expression for $\mathbf{V}_{AB}$ in the circuit below.



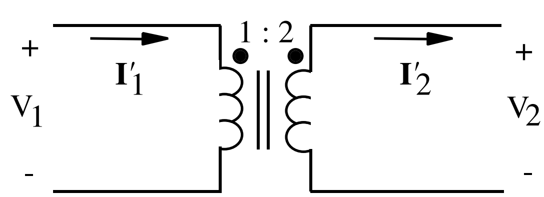

Balanced three-phase system.

$\mathbf{V}_{an} = 52 \angle 0°\text{V}$ $\mathbf{V}_{bn} = 52 \angle -120°\text{A}$ $z_{line} = j12\ \Omega$

**ans: a)** $z_{in} = 5\ \Omega$

**b)** $\mathbf{V}_{AB} \approx 234\angle -37.38°\ \text{V}$

**sol'n: (a)** Transformer is ideal. To distinguish currents in the transformer itself from other currents, we use a prime to denote the transformer currents. The current flowing into the dot on the primary side is $\mathbf{I}'_1$, and the current flowing out of the dot on the secondary side is $\mathbf{I}'_2$:

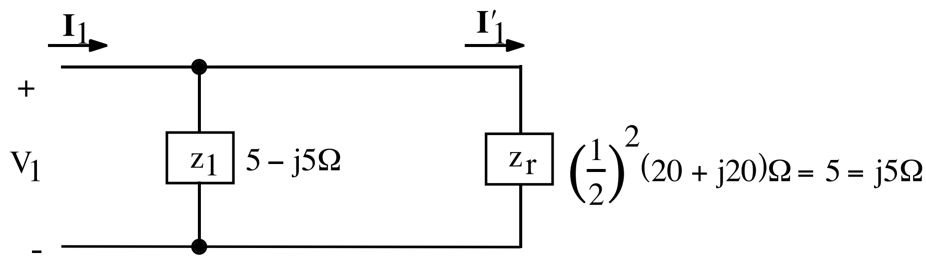



$$\frac{V_1}{V_2} = \frac{N_1}{N_2} \qquad \frac{\mathbf{I}'_1}{\mathbf{I}'_2} = \frac{N_2}{N_1}$$

Using the above model, we can derive the formula (or we can just look up the formula) for secondary impedance reflected into the primary:

$$z_r = \left(\frac{N_1}{N_2}\right)^2 z_2$$

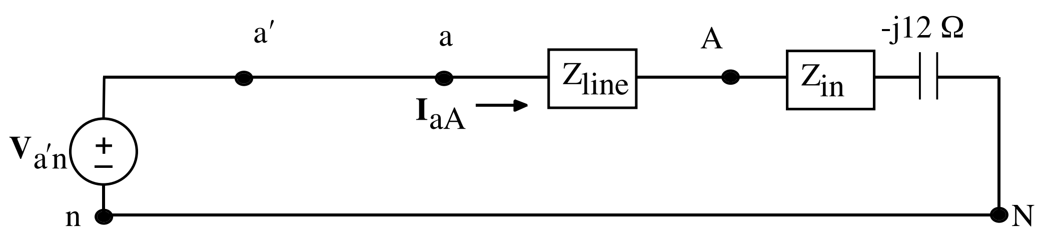
Our model, given $N_1/N_2 = 1/2$ turns ratio, is:




$$z = z_1 \| z_r = (5 - j5) \| (5 + j5)\Omega$$

$$z = \frac{(5 - j5)(5 + j5)}{5 - j5 + 5 + j5} = \frac{5^2 + 5^2}{10} = 5\Omega$$

**sol'n: (b)** Our first step is to convert our circuit to a Y – Y form so we can use a single-phase equivalent model. In this problem, the circuit is already in Y – Y form and we may draw the single-phase equivalent directly:



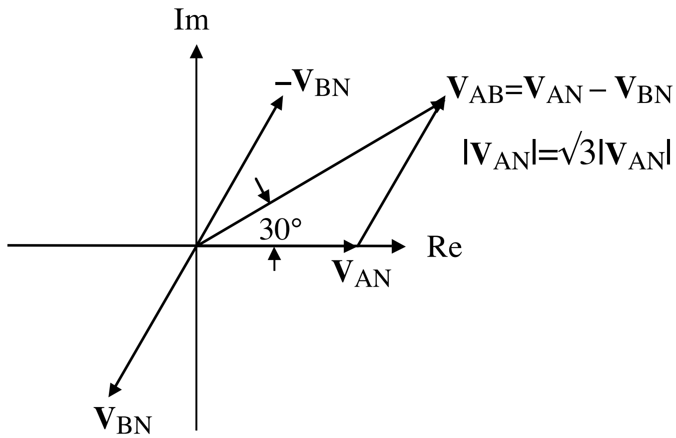

We find $\mathbf{V}_{AN}$ and then calculate $\mathbf{V}_{AB}$ using phasor diagrams. We obtain $\mathbf{V}_{AN}$ from the voltage divider formula:

$$\mathbf{V}_{AN} = \mathbf{V}_{a'n} \frac{z_{in} - j12\Omega}{z_{line} + z_{in} - j12\Omega}$$

$$\mathbf{V}_{AN} = 52\angle 0°V \frac{5\Omega - j12\Omega}{j12\Omega + 5\Omega - j12\Omega} = 52\angle 0°V \frac{5\Omega - j12\Omega}{5\Omega}$$

$$\mathbf{V}_{AN} = 52\angle 0°V \frac{13\angle -67.38°\Omega}{5\Omega}$$

We use a phasor diagram to relate $\mathbf{V}_{AN}$ to $\mathbf{V}_{AB}$. The diagram shows the relationship between $\mathbf{V}_{AN}$ and $\mathbf{V}_{AB}$, and we assume $\mathbf{V}_{AN}$ has phase angle zero so we can find the relative phase angle of $\mathbf{V}_{AB}$.




From the diagram, we deduce that

$$\mathbf{V}_{AB} = \mathbf{V}_{AN} \cdot \sqrt{3}\angle 30°$$

Plugging in the value of $\mathbf{V}_{AN}$ gives the numerical value of $\mathbf{V}_{AB}$.

$$\mathbf{V}_{AB} = 52\angle 0° V \frac{13\angle -67.38°\Omega}{5\Omega}\sqrt{3}\angle 30° = \frac{52 \cdot 13 \cdot \sqrt{3}}{5}\angle -37.38° V$$

$\mathbf{V}_{AB} \approx 234\angle{-37.38°}$ V