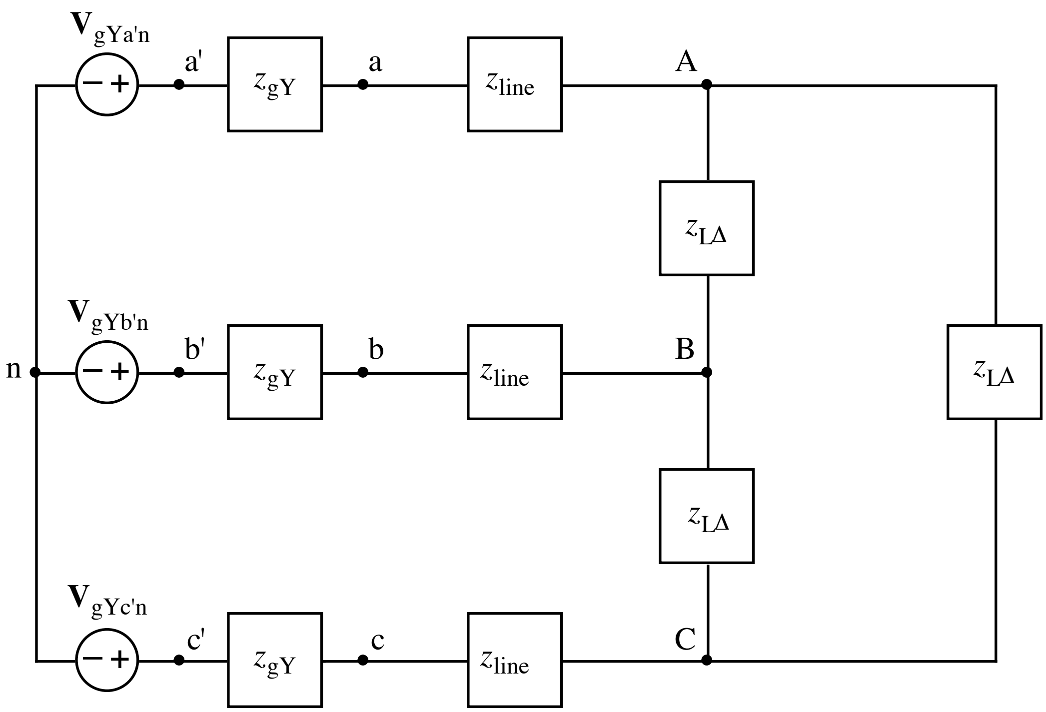
HW 7 prob 3 solution

Ex:



$V_{gYa'n} = 101\angle 0° \text{ V}$ $\qquad z_{gY} = 1 + j0.2\ \Omega$

$V_{gYb'n} = 101\angle{-120°} \text{ V}$ $\qquad z_{line} = 6 + j0.3\ \Omega$

$V_{gYc'n} = 101\angle{+120°} \text{ V}$ $\qquad z_{L\Delta} = 9 - j1.5\ \Omega$

a) Draw the single phase equivalent circuit.

b) Calculate $\mathbf{I}_{BC}$.

SOL'N: a) We obtain the single-phase equivalent by converting the source and load to Y configurations and using the A phase loop with a neutral line included in the system. The neutral line in a balanced 3-phase system carries no current and may be deleted without affecting the system. In the single-phase equivalent circuit, the neutral line carries nonzero current and completes the circuit for one phase. The line current, $\mathbf{I}_{aA}$, for the single-phase equivalent circuit is the same as the line current for the 3-phase system. Using $\mathbf{I}_{aA}$ and the single-phase voltages, $\mathbf{V}_{an}$ and $\mathbf{V}_{AN}$, and using 120° phase shifts, we can find any current or voltage for the original 3-phase system.

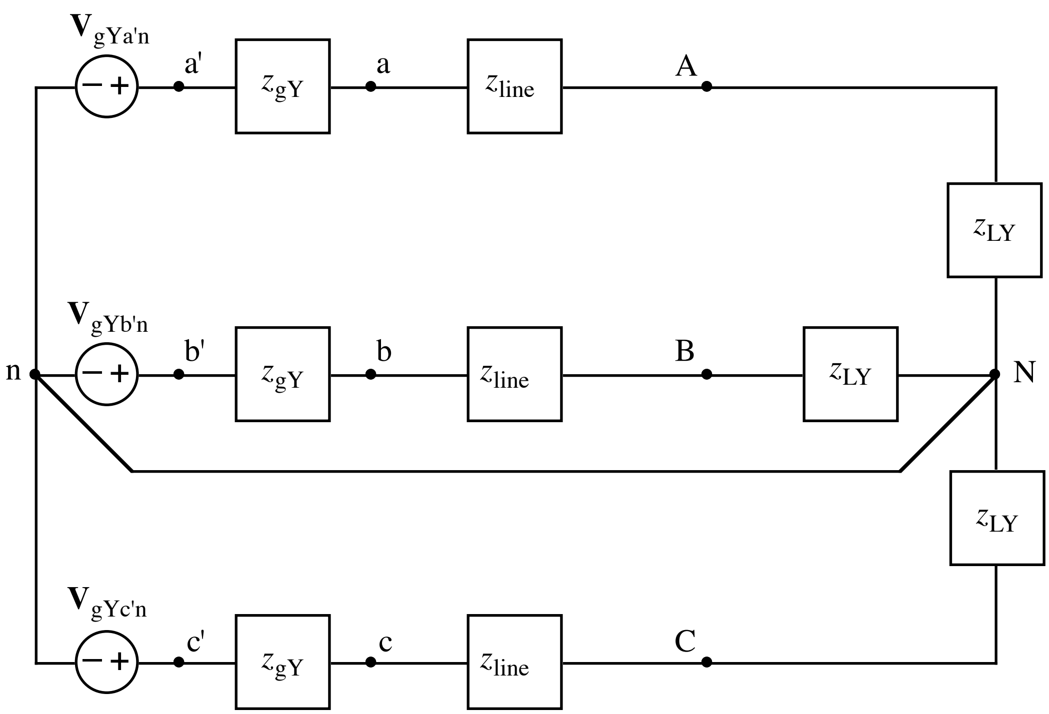
In this problem we need only convert the load to a Y configuration. We have the following relationship for z_{LY} and $z_{L\Delta}$:

$$z_{LY} = \frac{z_{L\Delta}}{3}$$

Using the value of $z_{L\Delta}$ given in the problem we have the following value for z_{LY}:

$$z_{LY} = \frac{9 - j1.5\ \Omega}{3} = 3 - j0.5\ \Omega$$

The circuit with the load converted to a Y configuration and a neutral line added is as follows:

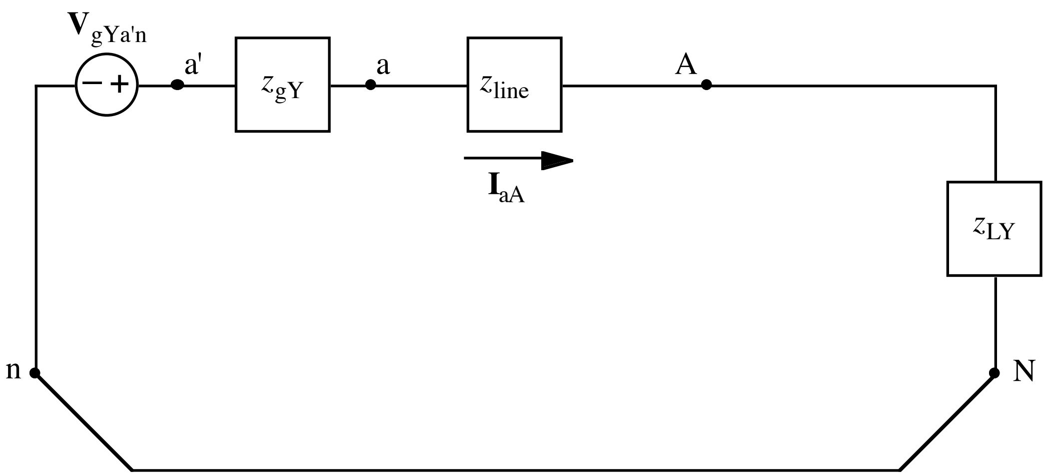


$V_{gYa'n} = 101\angle 0°$ V $\qquad z_{gY} = 1 + j0.2\ \Omega$

$V_{gYb'n} = 101\angle{-120°}$ V $\qquad z_{line} = 6 + j0.3\ \Omega$

$V_{gYc'n} = 101\angle{+120°}$ V $\qquad z_{LY} = 3 - j0.5\ \Omega$

The single-phase equivalent circuit is the A phase shown below:



$V_{gYa'n} = 101\angle 0°$ V

$z_{gY} = 1 + j0.2\ \Omega$

$z_{line} = 6 + j0.3\ \Omega$

$z_{LY} = 3 - j0.5\ \Omega$

b) $\mathbf{I}_{BC}$ is equal to $\mathbf{I}_{AB}$ shifted by –120°, and $\mathbf{I}_{AB}$ is found by finding $\mathbf{I}_{aA}$ and writing a current summation equation at the A node in the original 3-phase circuit. ($\mathbf{I}_{aA}$ is the same in the single-phase model as in the original circuit.)

NOTE: $\mathbf{I}_{aA}$ flows in the neutral line in this model, but it is cancelled out by the other currents flowing the complete circuit.

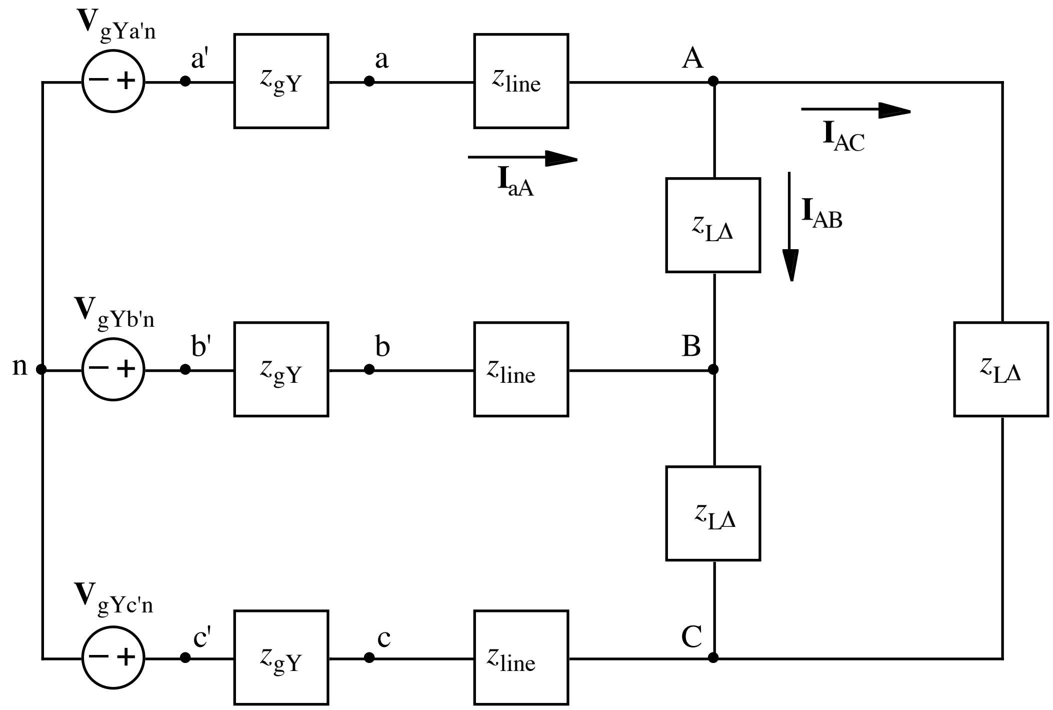
The value of $\mathbf{I}_{aA}$ is given by the voltage of the source divided by the total impedance in the single-phase equivalent circuit.

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{gYna}}{z_{gY} + z_{line} + z_{LY}} = \frac{101\angle 0°\ \text{V}}{1 + j0.2 + 6 + j0.3 + 3 - j0.5\ \Omega}$$

or

$$\mathbf{I}_{aA} = \frac{101\angle 0°}{10}\ \text{A} = 10.1\angle 0°\ \text{A}$$

Now we calculate the current sum at the A node.



$$\mathbf{I}_{\text{aA}} = \mathbf{I}_{\text{AB}} + \mathbf{I}_{\text{AC}}$$

If we reverse the direction of the measurement of $\mathbf{I}_{\text{AC}}$ to write our equation in terms of $\mathbf{I}_{\text{CA}}$, we change the sign of the value of the current. Equivalently, we shift the phase of the sinusoidal waveform by ±180°.

$$\mathbf{I}_{\text{AC}} = -\mathbf{I}_{\text{CA}} = \mathbf{I}_{\text{CA}} \cdot 1\angle -180°$$

We also have that $\mathbf{I}_{\text{CA}}$ is the same as $\mathbf{I}_{\text{AB}}$ but shifted by +120°.

$$\mathbf{I}_{\text{CA}} = \mathbf{I}_{\text{AB}} \cdot 1\angle 120°$$

Back substituting into the previous two equations, we have the following expression:

$$\mathbf{I}_{\text{aA}} = \mathbf{I}_{\text{AB}} + \mathbf{I}_{\text{AB}}\angle(120° - 180°) = \mathbf{I}_{\text{AB}} + \mathbf{I}_{\text{AB}}\angle -60°$$

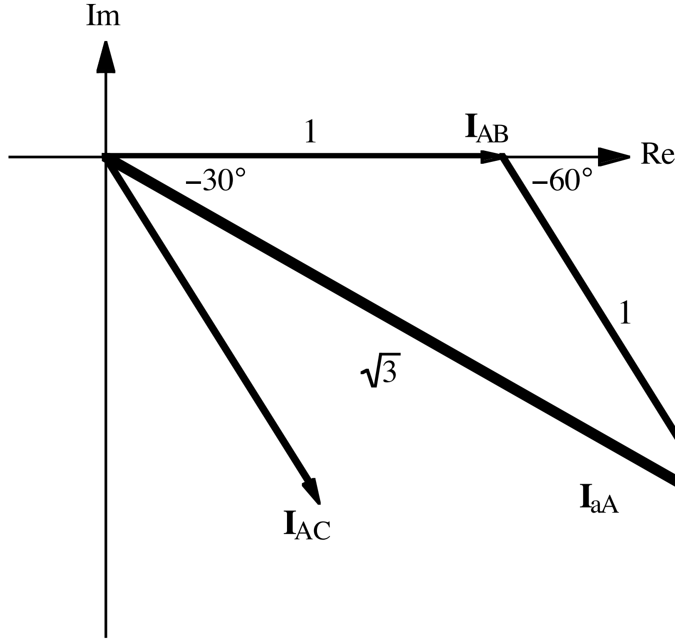
or

$$\mathbf{I}_{\text{aA}} = \mathbf{I}_{\text{AB}}(1 + 1\angle -60°)$$

To perform the addition, we can use mathematics.

$$1+1\angle-60° = 1+\frac{1}{2}-j\frac{\sqrt{3}}{2} = \frac{3}{2}-j\frac{\sqrt{3}}{2} = \sqrt{3}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right) = \sqrt{3}(1\angle-30°)$$

Alternatively, we may use a phasor diagram:



NOTE: When we draw the diagram, we may assume that $\mathbf{I}_{AB}$ is at 0° in order to find the relationship between $\mathbf{I}_{AB}$ and $\mathbf{I}_{aA}$. The equation yields the correct relationship because it expresses the relative values of the currents. If we used the correct angles in the diagram, the entire diagram would be rotated, but the relationship between values is the same.

From both approaches, we have the following result:

$$\mathbf{I}_{aA} = \mathbf{I}_{AB}\sqrt{3}(1\angle-30°)$$

Solving for $\mathbf{I}_{AB}$ in terms of $\mathbf{I}_{aA}$ we have the following:

$$\mathbf{I}_{AB} = \mathbf{I}_{aA}\frac{1}{\sqrt{3}}(1\angle+30°)$$

Substituting the value of $\mathbf{I}_{aA}$ from earlier, we obtain the numerical value of $\mathbf{I}_{AB}$:

$$\mathbf{I}_{AB} = 10.1 \cdot \frac{1}{\sqrt{3}}(1\angle + 30°) \text{ A} = 5.83\angle 30° \text{ A}$$

The value of $\mathbf{I}_{BC}$ is the value of $\mathbf{I}_{AB}$ shifted by –120°.

$$\mathbf{I}_{BC} = \mathbf{I}_{AB} \cdot 1\angle -120° = 5.83\angle -90° \text{ A}$$

NOTE: There are many ways we might write current summation equations for this problem. We could firrst shift signals by 120°, for example, and then sum the currents at the B node. All the approaches will yield the same answer, however.