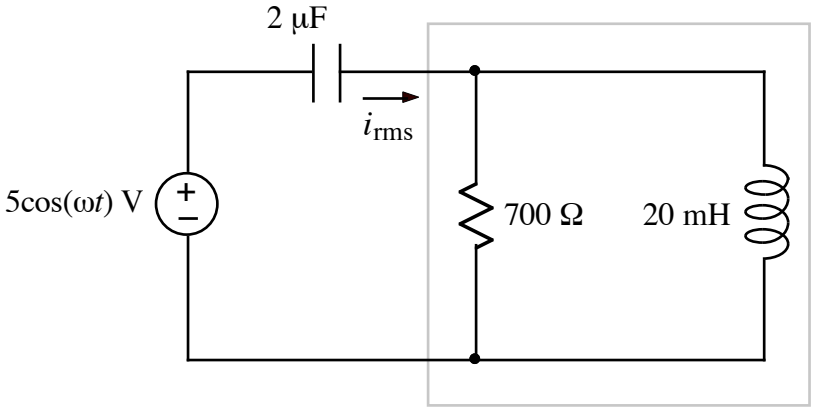
HW 7 prob 2 solution

EX:



a) Calculate $\mathbf{I}_{rms}$. Note: ω = 5 kr/s.

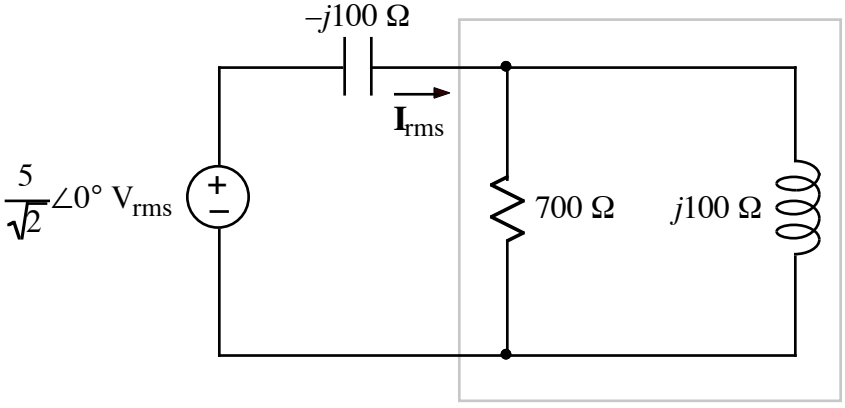
b) Calculate the complex power, *S*, for the components inside the box.

SOL'N: a) We first create an *s*-domain model. We have the following impedance values:

$$\frac{1}{j\omega C} = \frac{1}{j5\text{k r/s} \cdot 2\ \mu\text{F}} = -j100\ \Omega$$

$$j\omega L = j5\text{k r/s} \cdot 20\ \text{mH} = j100\ \Omega$$

Our *s*-domain model, with the rms value of the voltage source:



We use Ohm's law to calculate $\mathbf{I}_{rms}$:

$$\mathbf{I}_{rms} = \frac{\mathbf{V}_{rms}}{z_{tot}} = \frac{5/\sqrt{2}\ V_{rms}}{-j100 + 700 \parallel j100\ \Omega} = \frac{5/\sqrt{2}\ V_{rms}}{100\left(-j + \frac{j7}{7+j}\right)\Omega}$$

or

$$\mathbf{I}_{\text{rms}} = \frac{5/\sqrt{2}\ \text{V}_{\text{rms}}}{100\left(-j\frac{7+j}{7+j}+\frac{j7}{7+j}\right)\ \Omega} = \frac{5/\sqrt{2}}{100\dfrac{1}{7+j}}\ \text{A}_{\text{rms}}$$

or

$$\mathbf{I}_{\text{rms}} = \frac{50}{\sqrt{2}}(7+j)\ \text{mA}_{\text{rms}} = \frac{50}{\sqrt{2}}\sqrt{50}\angle\tan^{-1}\left(\frac{1}{7}\right)\text{mA}_{\text{rms}}$$

or

$$\mathbf{I}_{\text{rms}} = 50\sqrt{25}\angle 8.13^\circ\ \text{mA}_{\text{rms}}$$

b) We use the rms formulas for S:

$$S = \mathbf{V}_{\text{rms}}\mathbf{I}_{\text{rms}}^* = \mathbf{I}_{\text{rms}}z\mathbf{I}_{\text{rms}}^* = |\mathbf{I}_{\text{rms}}|^2 z$$

Using numerical values, we have the following:

$$S = \left(50\sqrt{25}\ \text{m}\right)^2 \cdot 100 \cdot 7\ \|\ j = \frac{25}{20^2}\cdot 100\cdot\frac{j7}{7+j}\ \text{VA}$$

or

$$S = \frac{25}{4}\cdot\frac{j7}{7+j}\cdot\frac{7-j}{7-j} = \frac{25}{4}\cdot\frac{7(1+j7)}{50} = \frac{7(1+j7)}{8}\ \text{VA}$$

In polar form, we have the following:

$$S = \frac{7\sqrt{50}}{8}\tan^{-1}\left(\frac{7}{1}\right) = \frac{7\sqrt{50}}{8}\angle 81.7^\circ\ \text{VA}$$

NOTE: Since S is a power quantity, there is no factor of $\sqrt{2}$ for rms after we have calculated S. A factor of 1/2 has, however, been embedded in the final answer via the product of rms quantities.