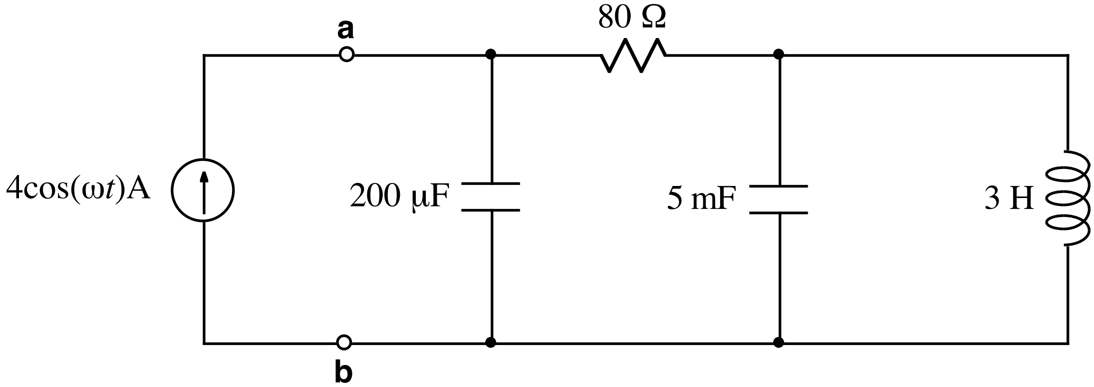
HW 7 prob 1 solution

Ex:



Note: $\omega = 10$ r/s.

Do the following for the impedance to the right of the a, b terminals:

a) Calculate complex power S = P + jQ.

b) Calculate average (or DC) power.

c) Calculate maximum instantaneous power.

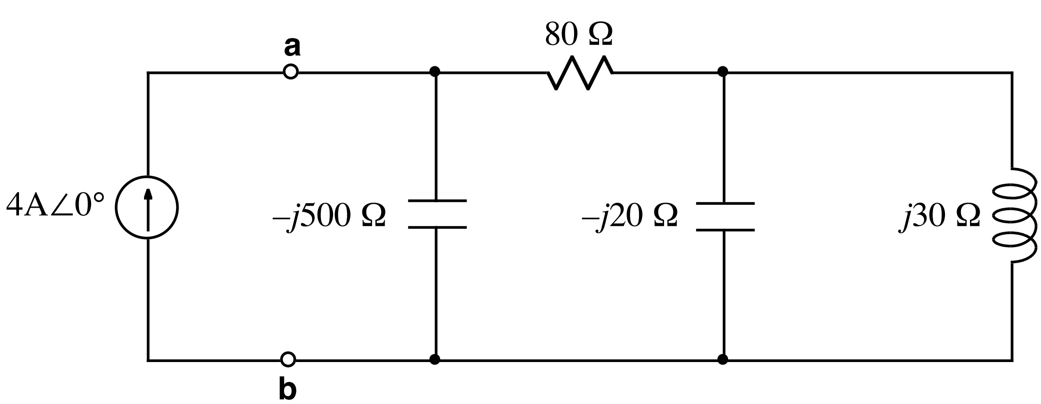
d) Sketch the power waveform, $p(t)$.

SOL'N: a) We first create an s-domain model. We have the following impedance values:

$$\frac{1}{j\omega C_1} = \frac{1}{j10\text{r/s} \cdot 200\mu\text{F}} = -j500\ \Omega \qquad \frac{1}{j\omega C_2} = \frac{1}{j10\text{r/s} \cdot 5\text{mF}} = -j20\ \Omega$$

$$j\omega L = j10\text{r/s} \cdot 3\text{H} = j30\ \Omega$$

Our s-domain model:



Now we calculate the impedance to the right of **a**, **b**.

$$z = -j500 \parallel (80 + -j20 \parallel j30)\ \Omega$$

or

$$z = 10 \cdot (-j50 \parallel (8 + -j2 \parallel j3))\ \Omega$$

or

$$z = 10 \cdot (-j50 \parallel (8 + \frac{6}{j}))\ \Omega$$

or

$$z = 10 \cdot (-j50 \parallel (8 - j6))\ \Omega$$

or

$$z = 20 \cdot (-j25 \parallel (4 - j3))\ \Omega$$

or

$$z = 20 \cdot \left(25 \cdot \frac{-3 - j4}{4 - j28}\right)\ \Omega$$

or

$$z = 5 \cdot \left(25 \cdot \frac{-3 - j4}{1 - j7} \cdot \frac{1 + j7}{1 + j7}\right)\ \Omega$$

or

$$z = 5 \cdot \left(25 \cdot \frac{25 - j25}{50}\right)\ \Omega$$

or

$$z = \frac{125 - j125}{2}\ \Omega$$

One formula for complex power involves only current and impedance:

$$S = \frac{1}{2}\boldsymbol{V}\boldsymbol{I}^* = \frac{1}{2}\boldsymbol{I}z \cdot \boldsymbol{I}^* = \frac{1}{2}|\boldsymbol{I}|^2 z$$

Using values from the problem, we find the numerical value of S:

$$S = \frac{1}{2}4^2 \frac{125 - j125}{2}\ \text{VA} = 500 - j500\ \text{VA}$$

b) The average DC power is equal to the real part of $S = P + jQ$, which is P:

$$P = 500 \text{ W}$$

NOTE: The value of P is, by coincidence, both the average power and the magnitude of the cosine portion of the AC part of the power waveform.

NOTE: The units we use when writing P are Watts. The units we use when writing S are VA. This equals Watts, but we write it in this alternate way so we can distinguish which quantity we are talking about when we see only a numerical value.

c) The power waveform is equal to a DC value of P plus a sinusoid whose phasor is S. It follows that the peak instantaneous value of power, $p_{\max}(t)$, is equal to P plus the magnitude of S:

$$p_{\max}(t) = P + |S| = P + \sqrt{P^2 + Q^2} = 500 + \sqrt{500^2 + 500^2} \text{ W}$$

or

$$p_{\max}(t) = 500 + 500\sqrt{2} \text{ W}$$

d) From the description in (c), we have the AC signal represented by S riding on top of the DC signal of height P. The AC signal is best represented in polar form when making a sketch:

$$\mathcal{L}^{-1}\{S = 500 - j500 \text{ VA}\} = \mathcal{L}^{-1}\{S = 500\sqrt{2}\angle -45° \text{ VA}\}$$
$$= 500\sqrt{2}\cos(2 \cdot 10t - 45°) \text{ W}$$

Adding the AC waveform to the DC offset, we have the following waveform for $p(t)$.

$$p(t) = 500 + 500\sqrt{2}\cos(20t - 45°) \text{ W}$$

NOTE: The AC part of the power has frequency 2ω. This arises from the trigonometric identity for the product of cosines.

For 2ω = 20, we have the following period:

$$T = \frac{1}{f} = \frac{2\pi}{2\omega} = \frac{2\pi}{20 \text{ r/s}} \approx 0.314 \text{ s} = 314 \text{ ms}$$

The plot of $p(t)$ is shown below.