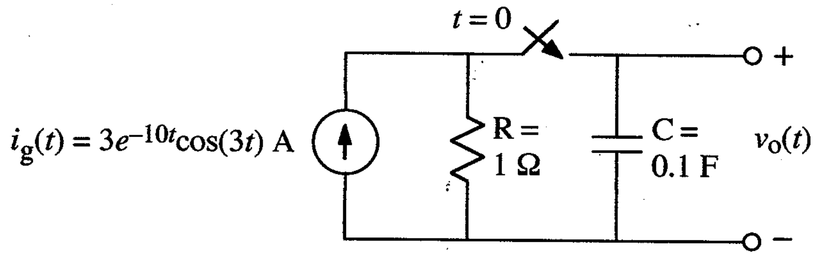
Ex:



Note: The initial voltage on the capacitor is $v_C(t = 0^-) = 3$ V measured with + on top.

a) Write the Laplace transform $I_g(s)$ of $i_g(t)$.

b) Write the Laplace transform $V_O(s)$ of $v_O(t)$. Be sure to include the effects of the initial voltage on the C.

c) Write a numerical time-domain expression for $v_O(t)$ where $t > 0$. Hint: plug in numbers and look for terms that cancel before computing partial fractions.

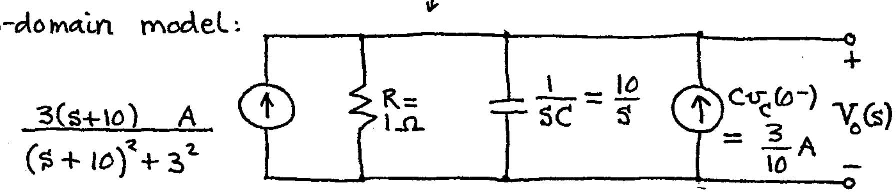
Sol'n: a) From a table of Laplace transforms:

$$\mathcal{L}\{i_g(t)\} = \mathcal{L}\{3e^{-10t}\cos(3t)\,A\} = \frac{3(s+10)}{(s+10)^2+3^2}\,A$$

b) It is convenient to use a parallel current source for initial conditions on the Capacitor.

switch acts like $u(t)$

s-domain model:



The output voltage is the total current times the total parallel resistance.

$$V_o(s) = \left(\frac{3(s+10)\,A}{(s+10)^2+3^2} + \frac{3}{10}A \right) R \| \frac{1}{sC}$$

Simplifying further, we have

$$R \| \frac{1}{sC} = R \cdot \left(\frac{1}{1} \| \frac{1}{sRC} \right) = \frac{R}{1+sRC} = \frac{1\,\Omega}{1+\frac{s}{10}} = \frac{10\,\Omega}{s+10}$$

$$\therefore V_o(s) = \left(\frac{3(s+10)\,A}{(s+10)^2+3^2} + \frac{3}{10}A \right) \frac{10\,\Omega}{s+10}$$

$$'' = \frac{30V}{(s+10)^2+3^2} + \frac{3V}{s+10}$$

$$V_o(s) = \frac{10V \cdot 3}{(s+10)^2+3^2} + 3V \cdot \frac{1}{s+10}$$

c) The above form for $V_o(s)$ is

$$V_o(s) = \frac{10V \cdot \omega}{(s+a)^2+\omega^2} + 3V \cdot \frac{1}{s+a} \quad \text{where } \begin{array}{l} a=10 \\ \omega=3 \end{array}$$

Using a table, we inverse transform $V_o(s)$ by inspection:

$$v_o(t) = [10V\,e^{-10t}\sin(3t) + 3V e^{-10t}]\,u(t)$$