(5 pts) c) Find $\lim_{t \to \infty} f(t)$ if $F(s) = \dfrac{3}{s[(s+4)^2 + 36]}$.

SOL'N: Apply the final value theorem:

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s\mathcal{L}\{f(t)\} = \lim_{s \to 0} sF(s)$$

We first factor out the highest power of s from the numerator and denominator and cancel out as many powers of s as possible:

$$sF(s) = \frac{s}{s} \cdot \frac{3}{[(s+4)^2 + 36]} = \frac{3}{[(s+4)^2 + 36]}$$

If there are pure powers of s remaining in the numerator or denominator, we may immediately conclude that the answer is zero or infinity, respectively.

Otherwise, as in the present case, we proceed to substitute $s = 0$ in the numerator and denominator to obtain our final result:

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s) = \frac{3}{4^2 + 36} = \frac{3}{52}$$

(5 pts) d) Plot the poles and zeros of F(s) in the s plane.

$$F(s) = \frac{s^2 + 8s + 16}{(s+8)(s^2 + 6s + 34)}$$

SOL'N: We factor the numerator and denominator:

$$F(s) = \frac{(s+4)^2}{(s+8)(s+3+j5)(s+3-j5)}$$

We plot the roots of the numerator, (i.e., the zeros), as **o**'s and the roots of the denominator, (i.e., the poles), as **x**'s.

Note that we use a small "2" to indicate the multiple zeros at $s = -4$.