ECE 2270
F 05

EXAM #3 Solution

THE
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1. (30 points)

(10 pts)   a)   Find $\mathcal{L}\left\{\int_0^t e^{-6\tau} u(6\tau)d\tau\right\}$.

SOL'N: Apply the scaling identity with $a = 6$ to $f(t) = e^{-t}u(t)$:

$$\mathcal{L}\{f(at)\} = \frac{1}{a}\mathcal{L}\{f(t)\}\Big|_{\text{replace } s \text{ with } \frac{s}{a}}$$

The result:

$$\frac{1}{6}\mathcal{L}\{e^{-6t}u(6t)\} = \frac{1}{6}\mathcal{L}\{e^{-t}u(t)\}\Big|_{\text{replace } s \text{ with } \frac{s}{6}}$$

$$\frac{1}{6}\mathcal{L}\{e^{-6t}u(6t)\} = \frac{1}{6}\frac{1}{(s+1)}\Big|_{\text{replace } s \text{ with } \frac{s}{6}}$$

The result we obtain when we simplify is the same as we would have if our original expression were written with $u(t)$ in place of $u(6t)$. This makes sense since scaling of t leaves $u(\ )$ unchanged.

$$\mathcal{L}\{e^{-6t}u(6t)\} = \frac{1}{6}\frac{1}{\left(\frac{s}{6}+1\right)} = \frac{1}{s+6}$$

To account for the integral, we multiply the previous answer by $1/s$:

$$\mathcal{L}\left\{\int_0^t e^{-6\tau} u(6\tau)d\tau\right\} = \frac{1}{s}\cdot\frac{1}{s+6} = \frac{1}{s(s+6)}$$

This problem is simple enough that we may easily check our answer by computing the integral before taking the Laplace transform:

$$\mathcal{L}\left\{\int_0^t e^{-6\tau} u(6\tau)d\tau\right\} = \mathcal{L}\left\{\frac{e^{-6t}}{-6}\Big|_0^t\right\} = \mathcal{L}\left\{\frac{1}{-6}(e^{-6t} - 1)\right\}$$

or

$$\mathcal{L}\left\{\int_0^t e^{-6\tau} u(6\tau)d\tau\right\} = \frac{1}{6}\left(\frac{1}{s} - \frac{1}{s+6}\right) = \frac{1}{s(s+6)} \quad \checkmark$$