

HW 5 prob 1.b solution

Ex: Find the Laplace transform of

$$\frac{d}{dt} \left[e^{-at} \sin(\omega t) u(t) \right]$$

SOL'N: Use the identity for derivatives:

$$\mathcal{L} \left\{ \frac{d}{dt} [f(t)u(t)] \right\} = sF(s) - f(0^-)$$

where

$$F(s) \equiv \mathcal{L}\{f(t)\} \equiv \int_{0^-}^{\infty} f(t)e^{-st} dt$$

From a lookup table of transform pairs, we have the following:

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^{-at} \sin(\omega t)\} = \frac{s + a}{(s + a)^2 + \omega^2}$$

For the initial value, we have zero:

$$f(0^-) = e^{-a \cdot 0^-} \sin(\omega \cdot 0^-) = 1 \cdot 0 = 0$$

Our final result:

$$\mathcal{L} \left\{ \frac{d}{dt} [e^{-at} \sin(\omega t) u(t)] \right\} = \frac{s(s + a)}{(s + a)^2 + \omega^2}$$