HW 5 prob 1.a solution

EX: Find the Laplace transform of

$$\cos(\omega t)u\left(t-\frac{1}{\omega}\right), \qquad \omega > 0$$

SOL'N: Use the identity for signals with a delayed turn-on and which are shifted in time:

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s), \qquad a > 0$$

where

$$F(s) \equiv \mathcal{L}\{f(t)\} \equiv \int_{0^-}^{\infty} f(t)e^{-st}dt$$

From the step function, $u\left(t-\frac{1}{\omega}\right)$, we identify a as $\frac{1}{\omega}$. To use the identity, we must write $\cos(\omega t)$ as a function of $t-\frac{1}{\omega}$:

$$\cos(\omega t) = \cos\left(\omega\left[\left(t-\frac{1}{\omega}\right)+\frac{1}{\omega}\right]\right)$$

We identify $f(t)$ by replacing $t-\frac{1}{\omega}$ with t:

$$f(t) = \cos\left(\omega\left[t+\frac{1}{\omega}\right]\right)$$

Now we apply a trigonometric identity to simplify this expression into something we can Laplace transform directly.

$$\cos\left(\omega\left[t+\frac{1}{\omega}\right]\right) = \cos(\omega t + 1) = \cos(\omega t)\cos(1) - \sin(\omega t)\sin(1)$$

NOTE: $\cos(1) = 0.5403$ and $\sin(1) = 0.8415$ are constants.

We lookup the Laplace transforms of $\cos(\omega t)$ and $\sin(\omega t)$ in a table:

$$\mathcal{L}\{\cos(\omega t)\} = \frac{s}{s^2+\omega^2} \quad \text{and} \quad \mathcal{L}\{\sin(\omega t)\} = \frac{\omega}{s^2+\omega^2}$$

Our final result:

$$\mathcal{L}\left\{\cos(\omega t)u\left(t-\frac{1}{\omega}\right)\right\} = e^{-\frac{1}{\omega}s}\left(\frac{\cos(1)s-\sin(1)\omega}{s^2+\omega^2}\right)$$