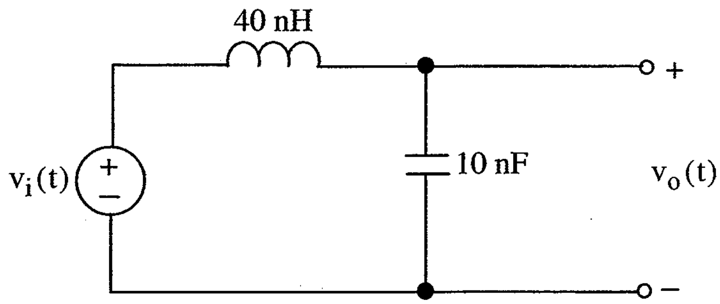
3. (30 points)



$$v_i(t) = \sum_{k=1}^{\infty} [a_k \cos(k\omega_o t) + b_k \sin(k\omega_o t)] V$$

where

$$a_k = -\frac{1}{\pi^2 k^2}$$

$$b_k = \frac{\cos\left(\frac{\pi}{2}k\right)}{k}$$

Write the time-domain expression of the fifth harmonic (i.e., k = 5) of $v_o(t)$.

Note: $\omega_o = 5M$ rad/s.

sol'n: For $k=5$, we have $a_k \equiv a_5 = \frac{-1\,V}{\pi^2 5^2} = \frac{-1\,V}{25\pi^2}$

$b_k \equiv b_5 = \frac{\cos\left(\frac{5\pi}{2}\right)}{5} = 0$

Thus, the phasor for the input signal's 5^{th} harmonic is $a_5 - j b_5 = a_5 = \frac{-1\,V}{25\pi^2} \equiv V_{i5}$

The 5^{th} harmonic of the output signal has phasor value $V_{o5} = V_{i5} \cdot H(j5\omega_o)$.

We have $H(s) = \frac{1/sC}{1/sC + sL} = \frac{1}{1+s^2LC}$

$$H(j5\omega_o) = \frac{1}{1+(j5\omega_o)^2 LC}$$

$$H(j5\omega_o) = \frac{1}{1 - 25\omega_o^2 LC}$$

$$'' = \frac{1}{1 - 25\cdot(5M)^2\cdot 40n \cdot 10n}$$

$$'' = \frac{1}{1 - \frac{1}{4}}$$

$$'' = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

$$V_{o5} = V_{i5}\, H(j5\omega_o) = -\frac{1\,V}{25\pi^2} \cdot \frac{4}{3}$$

$$V_{o5} = \frac{-4\ V}{75\pi^2}$$

$$v_{o5}(t) = \frac{4}{75\pi^2} \cos(25Mt + 180°)\ V$$

Note: Here, V_{i5} and $H(j5\omega_o)$ are real. In general, they are complex, and we get $V_{o5} = c + jd$. The inverse phasor of $c+jd$ would be
$$v_{o5}(t) = c\cdot\cos(5\omega_o t) - d\sin(5\omega_o t).$$