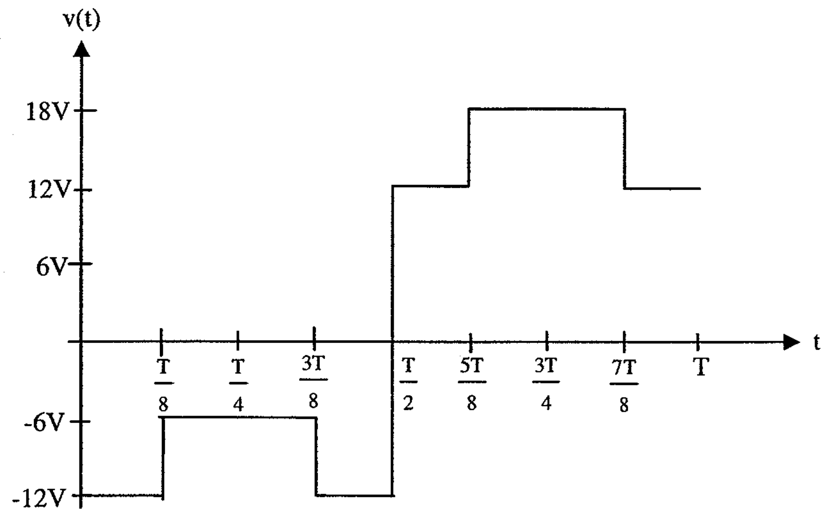
2. (40 points)



One period, T, of a function v(t) is shown above. The formula for v(t) is

$$v(t)=\begin{cases}-12V & 0<t<T/8\\ -6V & T/8<t<3T/8\\ -12V & 3T/8<t<T/2\\ 12V & T/2<t<5T/8\\ 18V & 5T/8<t<7T/8\\ 12V & 7T/8<t<T\end{cases}$$

Find the numerical value of the following coefficients of the Fourier series for v(t):

Pts

10 a. a_v

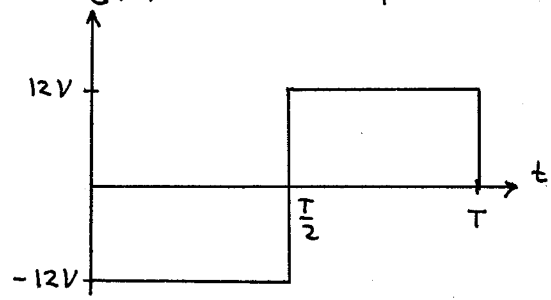
10 b. a_1

10 c. b_1

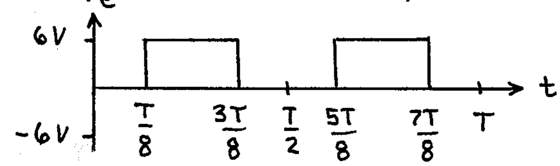
10 d. b_2

sol'n: a) Write v(t) as the sum of odd and even functions. The odd part will have no DC offset. We imagine spreading out the even part to a uniform height over period T to find a_v.

$v_o(t)$ odd part of $v(t)$



$v_e(t)$ even part of $v(t)$



By inspection, the average height of $v_e(t)$ is 3V.

$$\therefore \boxed{a_v = 3V} = \frac{1}{T}\int_0^T v(t)\,dt$$

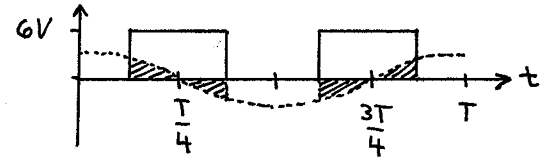
b) $$a_1 = \frac{2}{T}\int_0^T v(t)\cos(\omega_o t)\,dt \quad \text{where } \omega_o = \frac{2\pi}{T}.$$

Only $v_e(t)$ contributes to a_1 since an odd function like $v_o(t)$ has a_k's $= 0$.

$$\therefore a_1 = \frac{2}{T}\int_0^T v_e(t)\cos(\omega_o t)\,dt$$

We sketch $\int v_e(t)\cos(\omega_o t)$ = hatched area

$v_e(t)$ (vertical not to scale)



Areas cancel:

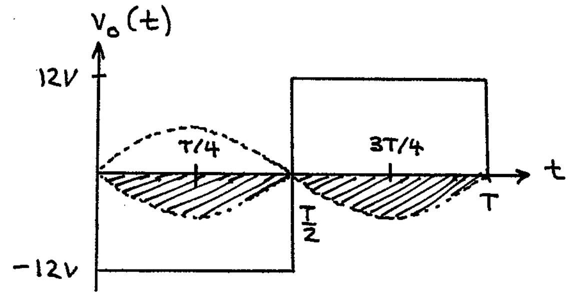
$$\therefore \boxed{a_1 = 0}$$

c) $$b_1 = \frac{2}{T}\int_0^T v(t)\sin(\omega_o t)\,dt$$

Only $v_o(t)$ contributes to b_1 since an even function like $v_e(t)$ has b_k's=0.

$$\therefore\ b_1 = \frac{2}{T}\int_0^T v_o(t)\sin(\omega_o t)\,dt$$

We sketch $\int v_o(t)\sin(\omega_o t)$, shown as hatch area, to look for symmetries.



We observe that we may integrate over 0 to T/2 and double the answer to get the total area. Or we may integrate over 0 to T/4 and quadruple the answer to get the total area.

Either approach is reasonable, but using T/2 as the upper limit may actually yield simpler values.

$$b_1 = \frac{2\cdot 2}{T}\int_0^{T/2} -12V\cdot\sin(\omega_o t)\,dt$$

where $\omega_o = \frac{2\pi}{T}$

$$b_1 = \frac{4}{\cancel{T}}(-12V)\frac{\left(-\cos\frac{2\pi}{T}t\right)}{2\pi/\cancel{T}}\Bigg|_0^{T/2} = \frac{24V}{\pi}\cdot(-1-1)$$

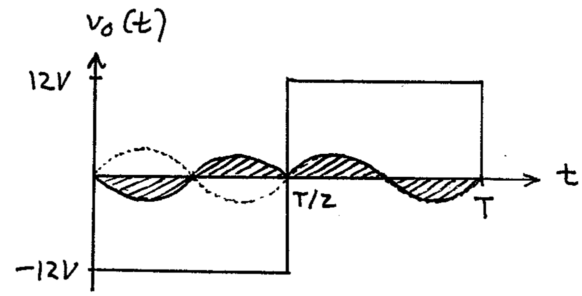
$$\boxed{b_1 = -48V/\pi}$$

d) $b_2 = \frac{2}{T}\int_0^T v(t)\sin(2\omega_0 t)\,dt$

Only $v_o(t)$ contributes to b_2 since $v_e(t)$ is an even function.

$$\therefore\ b_2 = \frac{2}{T}\int_0^T v_o(t)\sin(2\omega_0 t)\,dt$$

We sketch $\int v_o(t)\sin(2\omega_0 t)$ shown as *hatched area* to *find symmetries*.



The *hatched areas cancel out,* (*i.e., areas sum to zero*).

$$\therefore\ \boxed{b_2 = 0}$$