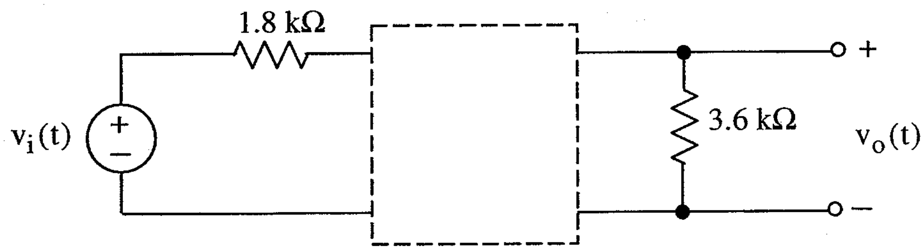
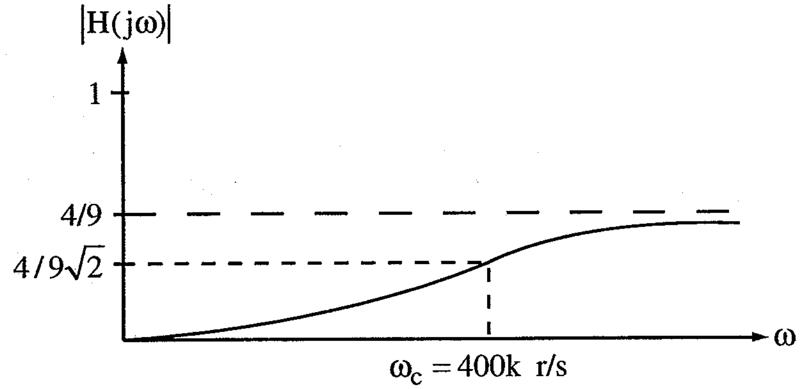
1. (30 points)





Using not more than one each R, L, and C, design a circuit to go in the dashed-line box that will produce the |H(jω)| vs. ω shown above, that is:

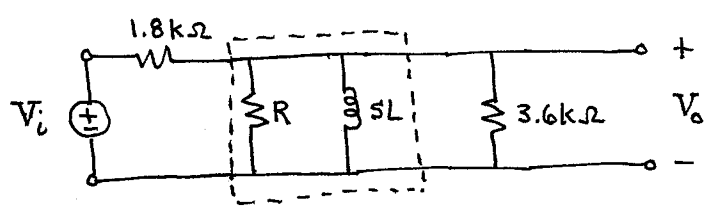
$$|H(j\omega)| = 0 \text{ at } \omega = 0$$

$$|H(j\omega)| = \frac{4}{9\sqrt{2}} \text{ at } \omega = 400\text{k rad/s}$$

$$|H(j\omega)| = \frac{4}{9} \text{ as } \omega \to \infty$$

soln: At least two soln's are possible.

I) Use an R and L in parallel across output:



The L shorts out for $\omega = 0 \Rightarrow V_o = 0, |H(j0)| = 0$.

As $\omega \to \infty$, $|H(j\infty)| \equiv \dfrac{V_o}{V_i} = \dfrac{R \| 3.6\,k\Omega}{R \| 3.6\,k\Omega + 1.8\,k\Omega}$ must be $\frac{4}{9}$,

(since L acts like open circuit).

An easy way to do the algebra to find R is to use ratios: $\frac{4}{9} = \frac{4}{4+5} = \frac{R \| 3.6\,k\Omega}{R \| 3.6\,k\Omega + 1.8\,k\Omega}$.

Thus, $\frac{R \| 3.6\,k\Omega}{1.8\,k\Omega} = \frac{4}{5}$. $R \| 3.6\,k\Omega = 1.8\,k\Omega \cdot \frac{4}{5}$

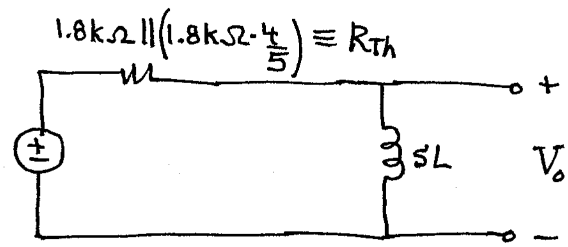
Factor out $1.8\,k\Omega$: $\frac{R}{1.8\,k\Omega} \| 2 = \frac{4}{5}$ or $R' \| 2 = \frac{4}{5}$

Now use conductance: $\frac{1}{R'} + \frac{1}{2} = \frac{5}{4}$ or $\frac{1}{R'} = \frac{3}{4}$

Thus, $R' = \frac{4}{3} \Rightarrow \frac{R}{1.8\,k\Omega} = \frac{4}{3}$, $R = \frac{4}{3}(1.8\,k\Omega) = 2.4\,k\Omega$.

To find ω_c, use a Thevenin equivalent for all the R's and V_i. From above, $R \| 3.6\,k\Omega = 1.8\,k\Omega \cdot \frac{4}{5}$.

$$\underbrace{V_i \, \frac{1.8\,k\Omega \cdot \frac{4}{5}}{1.8\,k\Omega \cdot \frac{4}{5} + 1.8\,k\Omega}}_{= \frac{4}{9} V_i}$$



This is now in standard form, so $\omega_c = \frac{R_{Th}}{L} = 400\,kr/s$.

$L = \frac{R_{Th}}{400\,k\,r/s}$

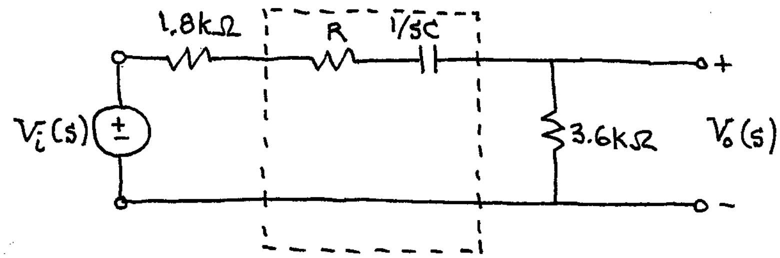
$R_{Th} = 1.8\,k\Omega \cdot 1 \| \frac{4}{5} = 1.8\,k\Omega \cdot \frac{\frac{4}{5}}{1 + \frac{4}{5}}$

$R_{Th} = 1.8\,k\Omega \cdot \frac{4}{9} = 0.8\,k\Omega$

$L = \frac{0.8\,k\Omega}{400\,kr/s} = \frac{0.8}{1k} \frac{(2.5)}{r/s}\,k\Omega = 2\,mH$

$\boxed{R = 2.4\,k\Omega}$ $\boxed{L = 2\,mH}$

II) Use an R and C in series with 1.8 kΩ.



For $\omega = 0$, the C acts like an open circuit. This gives $V_o(s=j0) = 0$ and $|H(s=j0)| = 0$.

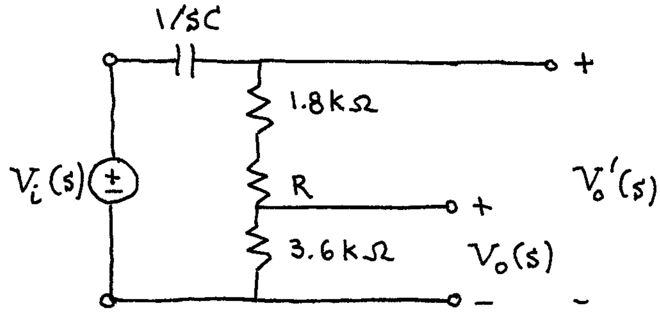
For $\omega \to \infty$, the C acts like a wire and $|H(j\omega)| = \dfrac{3.6\text{ k}\Omega}{1.8\text{ k}\Omega + R + 3.6\text{ k}\Omega}$ must be $\frac{4}{9}$.

or $|H(j\omega)| = \dfrac{4\ (0.9\text{ k}\Omega)}{9\ (0.9\text{ k}\Omega)} = \dfrac{3.6\text{ k}\Omega}{8.1\text{ k}\Omega}$

Thus, $1.8\text{ k}\Omega + R + 3.6\text{ k}\Omega = 8.1\text{ k}\Omega$.

$$\boxed{R = 2.7\text{ k}\Omega}$$

To find ω_c, rearrange components to put R's together, (without changing $H(s)$).



In this configuration we have a standard RC filter multiplied by a voltage divider.

$$H(s) = \frac{3.6k\Omega}{3.6\cancel{k\Omega + 1.8k\Omega} + R} \cdot \frac{\cancel{3.6k\Omega + 1.8k\Omega + R}}{\underbrace{3.6k\Omega + 1.8k\Omega + R}_{8.1k\Omega} + \frac{1}{sC}}$$

$$'' = \frac{3.6\,k\Omega}{8.1k\Omega + \frac{1}{sC}} = \frac{sC \cdot 3.6k\Omega}{1 + sC\;8.1k\Omega}$$

A better form of $H(s)$ for finding ω_c is $H(s) = \frac{4}{9} \cdot \frac{R'}{R' + \frac{1}{sC}}$ where $R' = 8.1k\Omega$.

or $H(s) = \frac{4}{9} \cdot \frac{sR'C}{1+sR'C}$

ω_c is where $|H(j\omega_c)| = \frac{1}{\sqrt{2}} \max_{\omega} |H(j\omega)|$

or $|H(j\omega_c)| = \frac{1}{\sqrt{2}} \frac{4}{9}$

or $\frac{|sR'C|}{|1+sR'C|} = \frac{1}{\sqrt{2}}$

The sol'n is $s = j\omega_c = j\frac{1}{R'C}$, as this

gives $|sR'C| = |j\frac{1}{R'C}R'C| = |j| = 1$

and $|1+sR'C| = |1+j\frac{1}{R'C}R'C| = |1+j| = \sqrt{2}$

So $\omega_c = \frac{1}{R'C}$ and $C = \frac{1}{R'\omega_c} = \frac{1}{8.1k\Omega\, 400kr/s}$

$$\boxed{C = 309\text{ pF}}$$