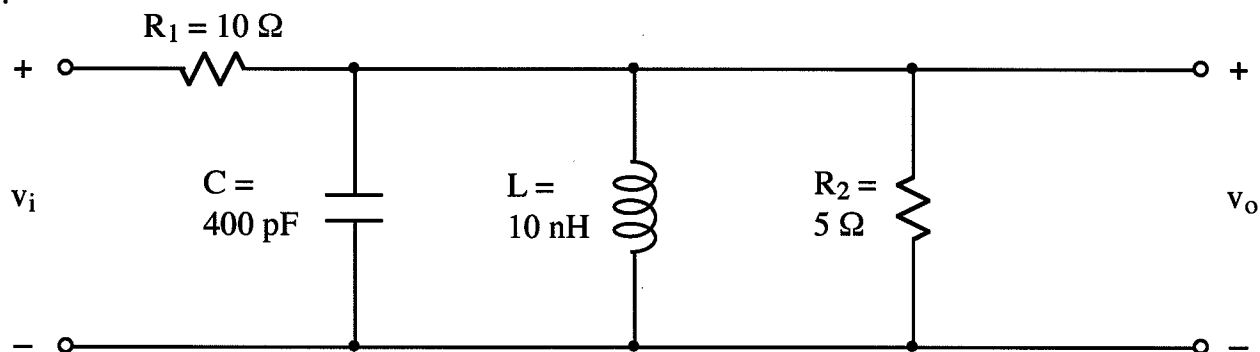


4.

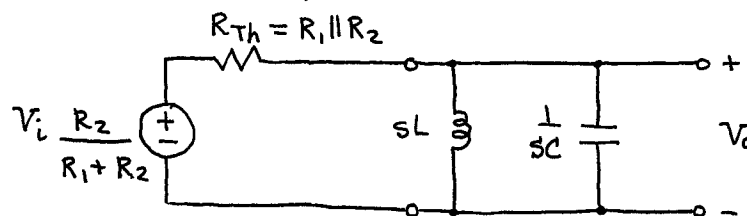


For the band-pass filter shown above, calculate the following quantities:

Hint: Use a Thevenin equivalent for the R's.

- a) ω_0
- b) ω_{C1} and ω_{C2}
- c) β
- d) Q

sol'n: a) We first move R_2 to the left side of L and C and replace v_i , R_1 , and R_2 with a Thevenin equiv.



Now we have a standard RLC filter but with a scaling factor of $\frac{R_2}{R_1 + R_2}$. This scaling

factor is real and scales the entire frequency response curve of $|H(s)|$ vs ω by the same factor. Thus $\omega_0^2 = \frac{1}{LC}$ still.

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{10n \cdot 400p} = \left(\frac{1}{2n}\right)^2$$

$$\omega_0 = \frac{1}{2n} = 500 \text{ Mr/s}$$

b) To find ω_{c1}, ω_{c2} we solve $|H(s)| = \frac{1}{\sqrt{2}} \max_{\omega} |H(s)|$.

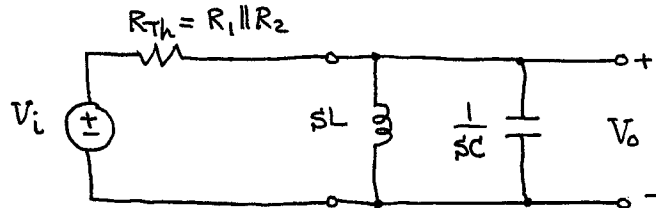
Because $\frac{R_2}{R_1+R_2}$ scales every frequency of $H(s)$

the same amount, we can ignore the $\frac{R_2}{R_1+R_2}$ and

solve $|H'(s)| = \frac{1}{\sqrt{2}} \max_{\omega} |H'(s)| = \frac{1}{\sqrt{2}}$ (see below)

$$\text{where } H'(s) = \frac{sL \parallel \frac{1}{sC}}{sL \parallel \frac{1}{sC} + R_{Th}} = \frac{1}{1 + \frac{R_{Th}}{sL \parallel \frac{1}{sC}}}$$

Note: this is the response of the following circuit:



Note: We can sketch $H'(s)$ by considering

$\omega=0$: $sL=0$ = wire, $\frac{1}{sC} = \infty$ = open
 $H'(j0)=0$ since V_o shorted by L

$\omega \rightarrow \infty$: $sL = \infty$ open, $\frac{1}{sC} = 0$ = wire
 $H'(j\infty)=0$ since V_o shorted by C

$\omega = \omega_0$: $sL = -1/sC \Rightarrow sL \parallel 1/sC = L/C / 0 = \infty$

$|H(s)|$ $H'(j\omega_0)=1$ since $sL \parallel 1/sC = \text{open}$



Back to $H'(s)$.

$$sL \parallel \frac{1}{sC} = \frac{L/C}{sL + \frac{1}{sC}} = \frac{L/C}{j(\omega L - \frac{1}{\omega C})}$$

$$H'(s) = \frac{1}{1 + \frac{R_{Th}}{\frac{L/C}{j(\omega L - \frac{1}{\omega C})}}} = \frac{1}{1 + jR_{Th}(\omega L - \frac{1}{\omega C}) \frac{L/C}{L/C}}$$

To find $\omega_{c1,2}$ we solve $|H'(s)| = \frac{1}{\sqrt{2}}$,

$$\text{or } \left| \frac{1}{1 + j \frac{R_{Th}(\omega L - \frac{1}{\omega C})}{L/C}} \right| = \frac{1}{\sqrt{2}}$$

$$\text{or } \sqrt{1^2 + \left[\frac{R_{Th}(\omega L - \frac{1}{\omega C})}{L/C} \right]^2} = \sqrt{2}$$

$$\text{or } \frac{R_{Th}(\omega L - \frac{1}{\omega C})}{L/C} = \pm 1$$

$$\text{or } R_{Th}(\omega L - \frac{1}{\omega C}) = \pm L/C$$

$$\text{or } \omega L - \frac{1}{\omega C} = \pm L/R_{Th}^C$$

$$\text{or } \omega^2 - \frac{1}{LC} = \pm \frac{1}{R_{Th}^C} \omega$$

$$\text{or } \omega^2 \pm \frac{1}{R_{Th}^C} \omega - \frac{1}{LC} = 0$$

$$\text{or } \omega_{d1,2} = \pm \frac{1}{2R_{Th}C} \pm \sqrt{\left(\frac{1}{2R_{Th}C}\right)^2 + \frac{1}{LC}}$$

Since $\omega_{d1,2} > 0$ we use $+\sqrt{\quad}$ only:

$$\omega_{d1,2} = \pm \frac{1}{2R_{Th}C} + \sqrt{\left(\frac{1}{2R_{Th}C}\right)^2 + \frac{1}{LC}}$$

Now for the numbers:

$$\frac{1}{2R_{Th}C} = \frac{1}{2 \cdot 5 \parallel 10 \cdot 400 \text{ p}} \text{ r/s}$$

$$= \frac{1}{2 \cdot 5 \cdot 1 \parallel 2 \cdot 400 \text{ p}} \text{ r/s}$$

$$= \frac{1}{2(5) \frac{2}{3} 400 \text{ p}} \text{ r/s}$$

$$= \frac{3}{2} \frac{1}{4 \text{ k p}} \text{ r/s}$$

$$\frac{1}{2R_{Th}C} = \frac{3}{8} \text{ G r/s}$$

$$\omega_0 = \frac{1}{2} \text{ G r/s} = \frac{4}{8} \text{ G r/s from earlier}$$

$$\omega_{d1,2} = \pm \frac{3}{8} + \sqrt{\left(\frac{3}{8}\right)^2 + \left(\frac{4}{8}\right)^2} \text{ G r/s}$$

$$= \pm \frac{3}{8} + \frac{5}{8} \text{ G r/s}$$

$$\omega_{d1,2} = \frac{1}{4} \text{ G and } 1 \text{ G r/s}$$

$$c) \quad \beta \equiv \text{bandwidth} = \omega_{c2} - \omega_{c1} = 2 \cdot \frac{1}{2R_{Th}C}$$

$$\beta = 2 \cdot \frac{3}{8} G \text{ r/s} = \frac{3}{4} G \text{ r/s}$$

$$d) \quad Q \equiv \text{quality factor} = \frac{\omega_0}{\beta} = \frac{\frac{1}{2} G \text{ r/s}}{\frac{3}{4} G \text{ r/s}} = \frac{2}{3}$$