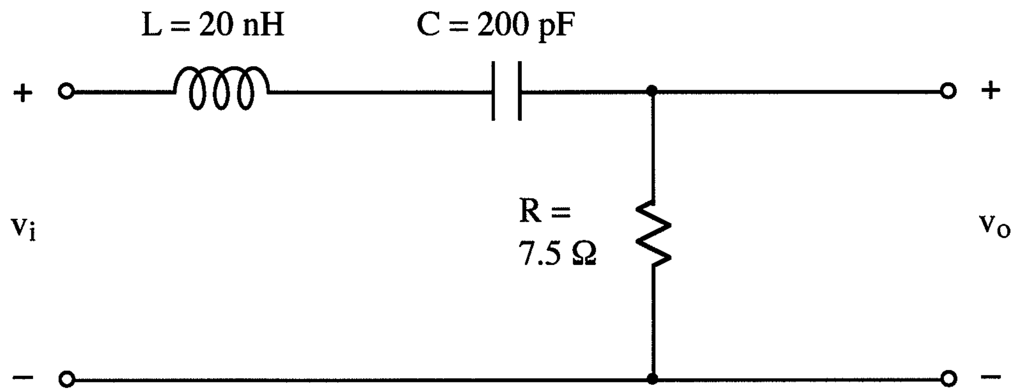
HOMEWORK #3 prob 3 soln

3.



For the band-pass filter shown above, calculate the following quantities:

a) ω_o

b) f_o

c) ω_{C1} and ω_{C2}

d) β and Q

soln: a) For this standard form of filter, the center frequency is $\omega_o = 1/\sqrt{LC}$.

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{20n \cdot 200p} = \left(\frac{1}{2n}\right)^2$$

Note: ω_o is where $|H(s)|$ = min or max. We accept that min or max of $|H(s)|$ occurs where $\omega = \omega_o$. We could prove this by finding $H(s)$.

$$\omega_o = \frac{1}{2n} \text{ r/s} = \frac{1}{2} \text{ G r/s or 500 M r/s}$$

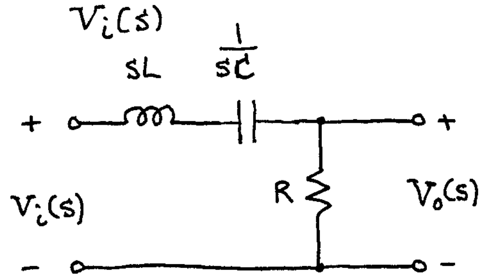
b) $f_o = \dfrac{\omega_o}{2\pi}$ since $\omega = 2\pi f$ in general

$f_o = \dfrac{500\,\text{M}}{2\pi}$ Hz $= 79.6$ kHz

c) We could use standard formulas for ω_{c1} and ω_{c2}, but the derivation is instructive.

$\omega_{c1,2}$ occur where $|H(s)| = \frac{1}{\sqrt{2}} \max_{\omega} |H(s)|$.

$H(s) \equiv \dfrac{V_o(s)}{V_i(s)}$ for frequency domain model:



$$H(s) = \frac{R}{R + sL + \frac{1}{sC}} = \frac{1}{1 + \dfrac{sL + \frac{1}{sC}}{R}}$$

$$H(j\omega) = \frac{1}{1 + \dfrac{j\omega L + \frac{1}{j\omega C}}{R}} = \frac{1}{1 + \dfrac{j\left(\omega L - \frac{1}{\omega C}\right)}{R}}$$

At ω_o we have $\omega L - \frac{1}{\omega C} = 0$ and $|H| = 1$.

This is the maximum value $|H|$ can have; this is the only frequency where the imaginary part of H is zero.

It follows that $\omega_{c1,2}$ solve $|H| = \frac{1}{\sqrt{2}} \cdot 1 = \frac{1}{\sqrt{2}}$.

$$|H(j\omega_c)| = \frac{1}{\left| 1 + j\frac{\left(\omega_c L - \frac{1}{\omega_c C}\right)}{R} \right|} = \frac{1}{\sqrt{2}}$$

or $$\left| 1 + j\frac{\left(\omega_c L - \frac{1}{\omega_c C}\right)}{R} \right| = \sqrt{2}$$

The sol'n is $$\frac{\omega_c L - \frac{1}{\omega_c C}}{R} = \pm 1$$

or $$\frac{L\omega_c^2 - \frac{1}{C}}{R} = \pm \omega_c$$

or $$L\omega_c^2 - \frac{1}{C} = \pm R\omega_c$$

or $$\omega_c^2 - \frac{1}{LC} \mp \frac{R}{L}\omega_c = 0$$

or $$\omega_c = \pm \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Since ω_c's > 0 and $\sqrt{}$ term $> \frac{R}{2L}$, we use

$$\omega_{c1,2} = \pm \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

Now for the numbers:

$$\frac{R}{2L} = \frac{7.5\,\Omega}{2 \cdot 20\,\text{nH}} = \frac{7.5}{40\text{n}} \cdot \frac{25}{25} = 187.5\ \text{M r/s}$$

$$\sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} = \sqrt{(187.5M)^2 + (500M)^2} \ r/s$$

$$= 534M \ r/s$$

Thus $\omega_{c1,2} = \pm 187.5\,M + 534\,M \ r/s$

$$\omega_{c1} = 346.5 \ Mr/s$$

$$\omega_{c2} = 721.5 \ Mr/s$$

d) Bandwidth $\beta = \omega_{c2} - \omega_{c1} = \frac{R}{L} = \frac{7.5}{20n} \ r/s$

$$\beta = \frac{7.5 \cdot 50}{20n \cdot 50} = 375 \ M\,r/s$$

Quality factor $Q = \frac{\omega_o}{\beta} = \frac{500\,M\,r/s}{375\,Mr/s} = \frac{4}{3}$