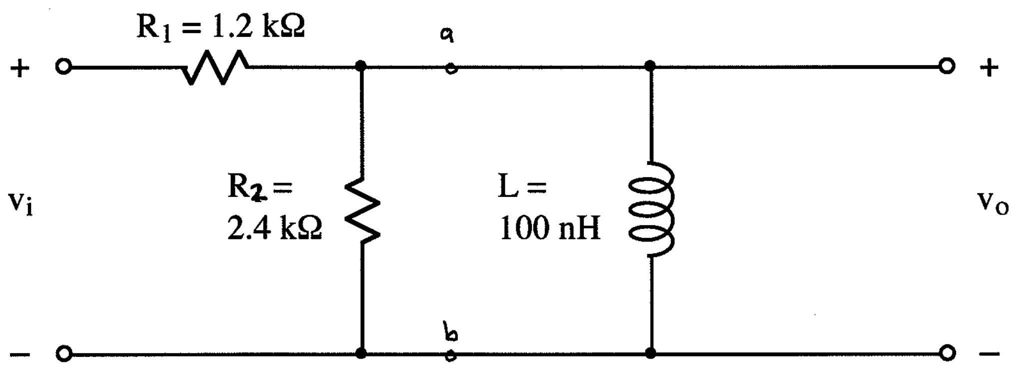
ECE 2270
F 06

HOMEWORK #3 prob 2 soln

THE UNIVERSITY OF UTAH

2.

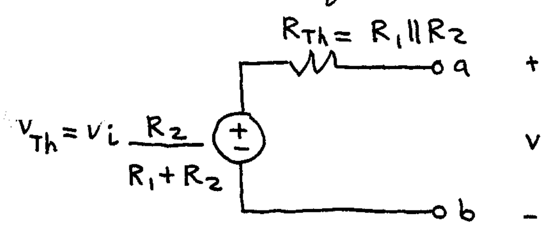


a) Determine the transfer function V_o/V_i. **Hint:** Use a Thevenin equivalent to reduce the two R's to a single R.

b) Plot $|V_o/V_i|$ versus ω.

c) Find the cutoff frequency, ω_c.

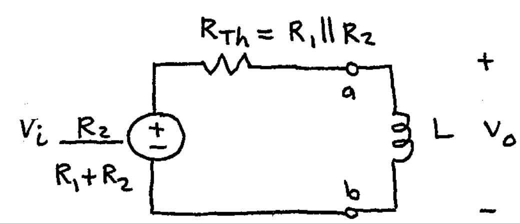
soln: a) The Thevenin equivalent of R_1, R_2, and V_i is



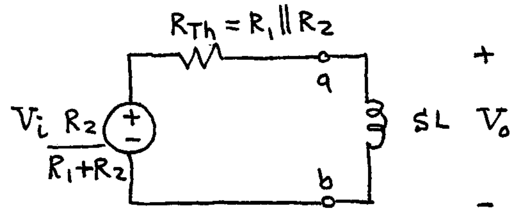
(Recall that $V_{Th} = V_{a,b\ \text{open circuit}}$, and

R_{Th} = R seen looking into a, b terminals with $V_i = 0V$ = wire.)

Now we add the L across a, b:



We transform to the s domain:



We use the voltage-divider formula for H(s):

$$H(s) \equiv \frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_1 + R_2} \, \frac{sL}{sL + R_{Th}}$$

$$H(s) = \frac{R_2}{R_1 + R_2} \, \frac{1}{1 + \dfrac{R_{Th}}{sL}}$$

b) For $\omega = 0$ we have

$$H(j0) = \frac{R_2}{R_1 + R_2} \, \frac{1}{1 + \dfrac{R_{Th}}{j \cdot 0 \cdot L}} = \frac{R_2}{R_1 + R_2} \, \frac{1}{1 + \infty}$$

$$H(j0) = \frac{R_2}{R_1 + R_2} \cdot 0 = 0$$

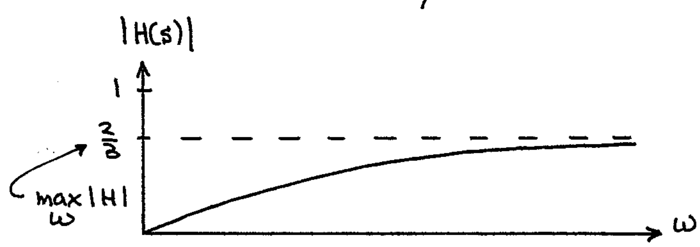
For $\omega \to \infty$ we have

$$H(j\infty) = \frac{R_2}{R_1 + R_2} \, \frac{1}{1 + \dfrac{R_{Th}}{j \cdot \infty \cdot L}} = \frac{R_2}{R_1 + R_2} \, \frac{1}{1 + 0}$$

$$H(j\infty) = \frac{R_2}{R_1 + R_2} = \frac{2.4k}{1.2k + 2.4k} = \frac{2}{3}$$

The value of $|H(s)| = \dfrac{R_2}{R_1+R_2} \dfrac{1}{\sqrt{1+\left(\dfrac{R_{Th}}{\omega L}\right)^2}}$

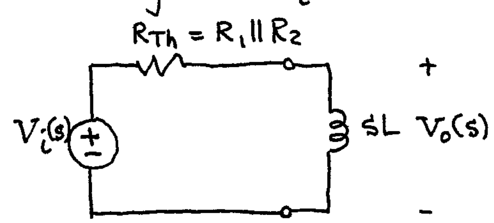
will increase steadily as ω increases.



c) ω_c occurs where $|H(s)| = \dfrac{1}{\sqrt{2}} \max_{\omega} |H(s)|$,

or $\dfrac{R_2}{R_1+R_2} \dfrac{1}{\left|1+\dfrac{R_{Th}}{j\omega_c L}\right|} = \dfrac{1}{\sqrt{2}} \dfrac{R_2}{R_1+R_2}$

We observe that the $\dfrac{R_2}{R_1+R_2}$ cancels out, making this the same problem as finding ω_c for the following filter that lacks the scaling of V_i:



For this filter, we have $H'(s) = \dfrac{sL}{sL+R_{Th}} = \dfrac{1}{1+\dfrac{R_{Th}}{sL}}$.

We now find ω_c for this filter:

$$|H'(s)| = \frac{1}{\left|1 + \frac{R_{Th}}{j\omega_c L}\right|} = \frac{1}{\sqrt{2}}$$

or $$\left|1 + \frac{R_{Th}}{j\omega_c L}\right| = \sqrt{2}$$

or $$\sqrt{1^2 + \left(\frac{R_{Th}}{\omega_c L}\right)^2} = \sqrt{2}$$

or $$\frac{R_{Th}}{\omega_c L} = \pm 1$$

Since $\omega_c > 0$ we solve $\frac{R_{Th}}{\omega_c L} = 1$

or

$$\omega_c = \frac{R_{Th}}{L} = \frac{R_1 \| R_2}{L} = \frac{1.2k\Omega \| 2.4k\Omega}{100\,nH}$$

$$= \frac{1.2k\Omega \cdot 1\|2}{100\,nH} = \frac{0.8k}{100\,n}\ r/s$$

$$\omega_c = 8\ G\ r/s$$