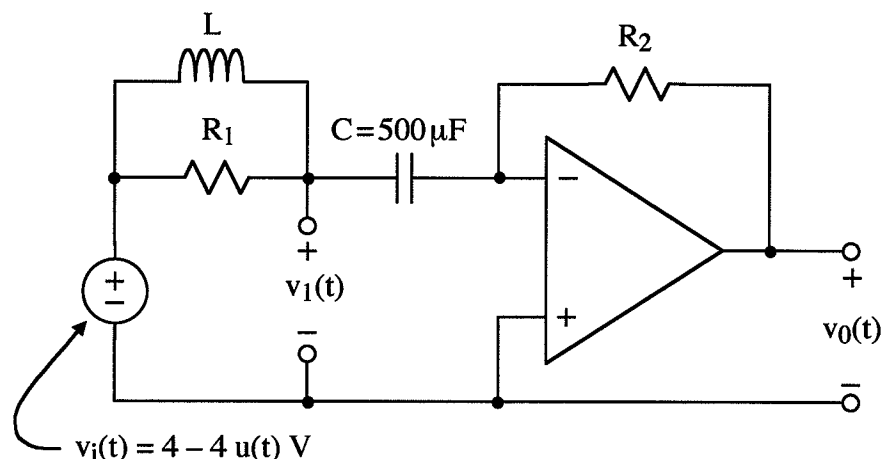


Ex:



The Laplace transform of $v_0(t)$ for the above circuit is as follows:

$$V_0(s) = 4V \cdot \frac{R_2}{R_1} \cdot \frac{s + \frac{R_1}{L}}{s^2 + \frac{1}{R_1 C}s + \frac{1}{LC}}$$

Choose numerical values for R_1 and L to make

$$v_1(t) = v_m e^{-\alpha t} \cos(\beta t + \varphi)$$

where $\alpha = \beta = 100 \text{ rad/s}$.

sol'n: We first observe that $\cos(\beta t + \varphi)$ is the same as a cosine of frequency β plus a sine of frequency β :

$$\cos(\beta t + \varphi) = \cos(\varphi) \cos(\beta t) - \sin(\varphi) \sin(\beta t)$$

Thus, we may rewrite $v_1(t)$ as follows:

$$v_1(t) = v_m \cos(\varphi) e^{-\alpha t} \cos(\beta t) - v_m \sin(\varphi) e^{-\alpha t} \sin(\beta t)$$

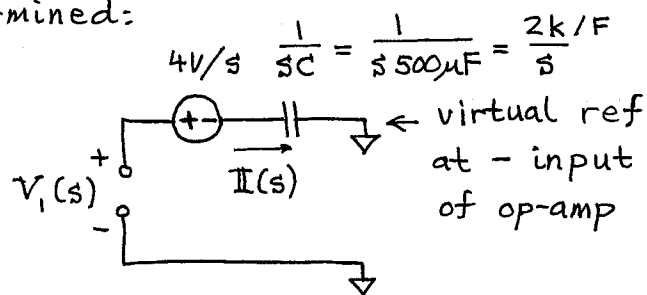
Taking the Laplace transform, we have

$$\mathcal{L}\{v_1(t)\} = V_1(s) = v_m \cos(\varphi) \frac{s + \alpha}{(s + \alpha)^2 + \beta^2} - v_m \sin(\varphi) \frac{\beta}{(s + \alpha)^2 + \beta^2}$$

From the circuit diagram, we see that $V_i(s)$ is the s-domain voltage across C.

The initial condition on C, found using $V_C(0^+) = V_C(0^-)$, is $V_C(t=0^-) = 4V$ (from $V_i(t < 0) = 4V$).

This yields the following circuit for $V_i(s)$ in which $I(s)$ remains to be determined:



From the circuit diagram in the problem statement, we have current $I(s)$ determining $V_o(s)$:

$$V_o(s) = -I(s)R_2$$

$$\text{or } I(s) = \frac{-V_o(s)}{R_2}$$

Using the expression for $V_o(s)$ given in the problem statement, we have the following expression for $I(s)$:

$$I(s) = -\frac{4V}{R_1} \frac{s + R_1/L}{s^2 + \frac{1}{R_1C}s + \frac{1}{LC}}$$

Now we can write the following eq'n:

$$\begin{aligned}
 V_1(s) &= \frac{4V}{s} + I(s) \cdot \frac{1}{sC} \\
 &= \frac{4V}{s} + \frac{-4V}{R_1} \frac{s + R_1/L}{s^2 + \frac{1}{R_1 C} s + \frac{1}{LC}} \cdot \frac{1}{sC} \\
 &= \frac{4V \cdot R_1 C (s^2 + s/R_1 C + 1/LC) - 4V(s + R_1/L)}{s \cdot R_1 C (s^2 + s/R_1 C + 1/LC)} \\
 &= \frac{4V \cdot R_1 C s^2}{s \cdot R_1 C (s^2 + s/R_1 C + 1/LC)} \\
 &= \frac{4V \cdot s}{s^2 + \frac{1}{R_1 C} s + \frac{1}{LC}}
 \end{aligned}$$

To match this to the symbolic form of $V_1(s)$ given earlier, we must have the same denominator:

$$\begin{aligned}
 s^2 + \frac{1}{R_1 C} s + \frac{1}{LC} &= (s + \alpha)^2 + \beta^2 \\
 " &= s^2 + 2\alpha s + \alpha^2 + \beta^2
 \end{aligned}$$

Note: We must also match numerators,
 $4V \cdot s = V_m \cos(\varphi)(s + \alpha) - V_m \sin(\varphi)\beta$,
 but this will be possible given
 arbitrary V_m and φ .

By matching the denominators, we have
 the following eq'ns:

$$\frac{1}{R_1 C} = 2\alpha \quad \text{where } \alpha = 100 \text{ rad/s}$$

$$C = 500 \mu\text{F}$$

$$\frac{1}{LC} = \alpha^2 + \beta^2 \quad \text{where } \beta = 100 \text{ rad/s}$$

Solving for R_1 and L , we have the following:

$$R_1 = \frac{1}{2\alpha \cdot C} = \frac{1}{2(100) \cdot 500 \mu} = 10 \Omega$$

$$L = \frac{1}{C(\alpha^2 + \beta^2)} = \frac{1}{500 \mu (100^2 + 100^2)} = 100 \text{ mH}$$