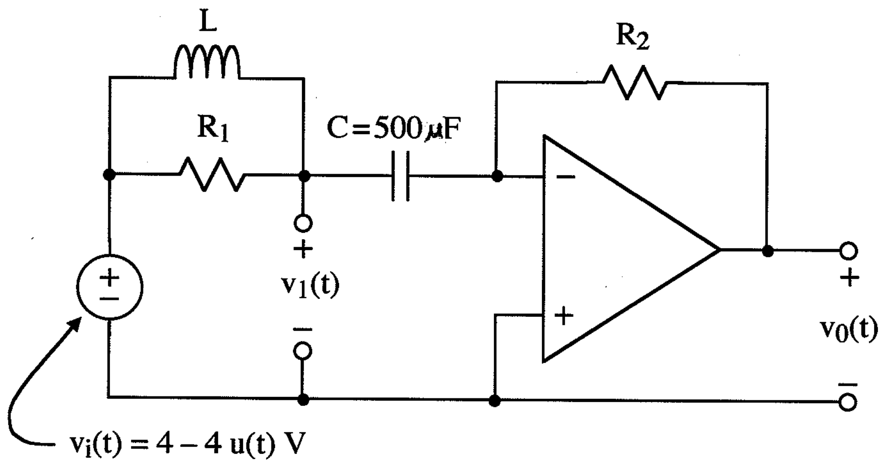
Ex:



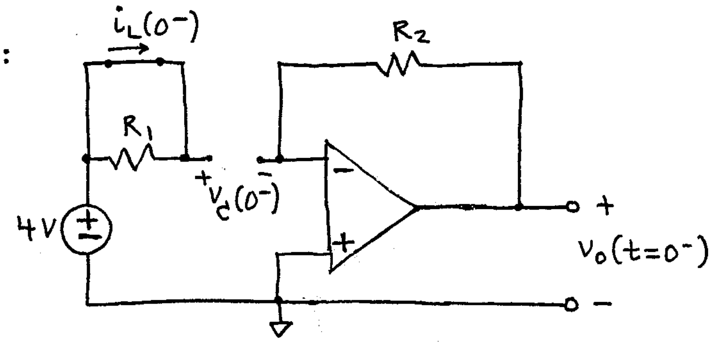
Find a symbolic expression for $\mathbf{V}_O(s)$ in terms of not more than R_1, R_2, L, C, and constants.

Soln: To find the Laplace transformed circuit elements, we first find initial conditions for L and C: $i_L(0^-)$ and $v_C(0^-)$.

For a long time before $t=0$, $v_i(t) = 4V$.

For this DC input, the circuit will reach an equilibrium with constant currents and voltages. Thus, derivatives di_L/dt and dv_C/dt equal zero. This, in turn, means $v_L = 0$ and $i_C = 0$. So L = wire and C = open.

$t=0^-$:



The op-amp has negative feedback that acts to keep $v_- \doteq v_+$. Thus, we have 0V at the inputs of the op-amp.

If we consider a voltage passing through the 4V source, L = wire, C = open, and across the op-amp inputs = 0V drop, we have

$$v_C(0^-) = 4V.$$

Since C = open, we also have

$$i_L(0^-) = 0A.$$

Now we Laplace transform the input voltage to obtain a model for the s-domain.

$$\mathcal{L}\{4 - 4u(t)\} = \mathcal{L}\{4\} - \mathcal{L}\{4u(t)\}$$

$$= \frac{4}{s} - \frac{4}{s}$$

$$= 0$$

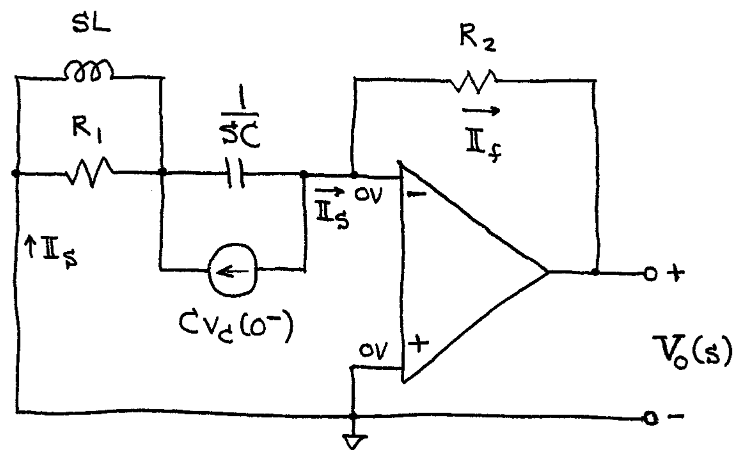
Thus, $V_i(s) = 0V$ which is wire.

Using a parallel current source for initial conditions on the L, we have $0/s = 0A$ = open. Thus, we may leave out this source.

Using a series voltage source for initial conditions on C, we would have $4V/s$. Since we have a virtual reference at v_-, however, C is effectively in parallel with R_1 and C.

Thus, we use a parallel current source for initial conditions on C. The value of the current source is $Cv_c(0^-) = C\cdot 4V$. The direction of current is chosen to place charge instantly on C that will yield 4V at time 0^+ across C. (Note that, in the time-domain, $C\cdot 4V$ corresponds to $C\cdot 4V\cdot \delta(t)$.)

s-domain model:



To solve this op-amp circuit, we observe that $v_+ = 0V$ and $v_- = v_+ = 0V$.

Since no current flows into the op-amp, we also have I_s flowing toward the − input from the left is the same as current I_f flowing in R_2. We write equations for I_s and I_f using $v_- = 0V$. Then we set $I_s = I_f$ and solve for $V_o(s)$.

We observe that I_s is the current flowing in R_1 and sL, and we use a current divider eq'n.

$$\mathbb{I}_s = -Cv_c(0^-)\frac{1/sC}{1/sC + R_1 \| sL}$$

$$= -Cv_c(0^-)\frac{1}{1 + sC \cdot R_1 \| sL}$$

$$= -Cv_c(0^-)\frac{1 + R_1/sL}{1 + \frac{R_1}{sL} + sR_1C}$$

$$= -Cv_c(0^-)\frac{sL + R_1}{sL + R_1 + s^2 R_1 LC}$$

$$\mathbb{I}_s = -\cancel{C}v_c(0^-)\frac{1}{R_1\cancel{C}} \cdot \frac{s + R_1/L}{s^2 + \frac{1}{R_1C}s + \frac{1}{LC}}$$

or
$$\mathbb{I}_s = -\frac{4V}{R_1}\frac{s + R_1/L}{s^2 + \frac{1}{R_1C}s + \frac{1}{LC}}$$

For $\mathbb{I}_f$, we have

$$\mathbb{I}_f = \frac{0V - V_o(s)}{R_2} = -\frac{V_o(s)}{R_2}$$

Equating $\mathbb{I}_s$ and $\mathbb{I}_f$ and solving for $V_o(s)$ yields

$$V_o(s) = -R_2\mathbb{I}_s$$

$$V_o(s) = 4V \cdot \frac{R_2}{R_1} \cdot \frac{s + R_1/L}{s^2 + \frac{1}{R_1C}s + \frac{1}{LC}}$$