TOOL: The following transform pair is useful for determining the inverse Laplace transform when coefficients and roots are complex:

$$\mathcal{L}\left[e^{-at}(2c\cos\omega t + 2d\sin\omega t)u(t)\right] = \frac{c+jd}{s+a+j\omega} + \frac{c-jd}{s+a-j\omega}$$

or

$$\mathcal{L}\left[e^{-at}(2\mathrm{Re}[A]\cos\omega t + 2\mathrm{Im}[A]\sin\omega t)u(t)\right] = \frac{A}{s+a+j\omega} + \frac{A^*}{s+a-j\omega}$$

If we express the sum of cos() and sin() as a cosine with a phase shift, we obtain the following result:

$$\mathcal{L}\left[e^{-at}2\sqrt{c^2+d^2}\cos\left(\omega t + \tan^{-1}\frac{d}{c}\right)u(t)\right] = \frac{c+jd}{s+a+j\omega} + \frac{c-jd}{s+a-j\omega}$$

or

$$\mathcal{L}\left[e^{-at}2|A|\cos\left(\omega t + \tan^{-1}\frac{\mathrm{Im}[A]}{\mathrm{Re}[A]}\right)u(t)\right] = \frac{A}{s+a+j\omega} + \frac{A^*}{s+a-j\omega}$$

DERIV: We begin with a partial fraction expansion with complex coefficients and complex roots. The coefficients will be complex conjugates, and we write the coefficients as $c + jd$ and $c - jd$ where c and d are real.

$$F(s) = \frac{c+jd}{s+a+j\omega} + \frac{c-jd}{s+a-j\omega}$$

We use a common denominator to identify terms that correspond to a decaying cosine and decaying cosine, (see below):

$$F(s) = \frac{(c+jd)(s+a-j\omega) + (c-jd)(s+a+j\omega)}{(s+a)^2+\omega^2}$$

A number of terms cancel in the numerator:

$$F(s) = \frac{2c(s+a)+2d\omega}{(s+a)^2+\omega^2} = \frac{2c(s+a)}{(s+a)^2+\omega^2} + \frac{2d\omega}{(s+a)^2+\omega^2}$$

The first term on the right corresponds to a decaying cosine waveform in the time-domain, and the second term on the right corresponds to a decaying sine waveform in the time-domain:

$$\mathcal{L}\left[e^{-at}(2c\cos\omega t + 2d\sin\omega t)u(t)\right] = \frac{2c(s+a)}{(s+a)^2+\omega^2} + \frac{2d\omega}{(s+a)^2+\omega^2}$$

To express this in terms of a cosine with a phase shift, we use trigonometric identities:

$$K\cos(\omega t + \theta) = K\cos(\omega t)\cos\theta - K\sin(\omega t)\sin\theta$$

It follows that the following equations must hold:

$$K\cos\theta = 2c \text{ and } K\sin\theta = 2d$$

Squaring both terms and summing yields the following result:

$$K^2 = K^2\cos^2\theta + K^2\sin^2\theta = 2^2(c^2+d^2)$$

or

$$K = 2\sqrt{c^2+d^2}$$

Taking the ratio of the cos() and sin() equations yields the following result:

$$\tan\theta = \frac{K\sin\theta}{K\cos\theta} = \frac{\sin\theta}{\cos\theta} = \frac{2d}{2c} = \frac{d}{c}$$

or

$$\theta = \tan^{-1}\frac{d}{c}$$