

**TOOL:** The following list of identities allows one to generate complicated Laplace transforms from basic Laplace transforms. Proofs follow from the definition of the Laplace transform:

$$\mathcal{L}[v(t)] \equiv V(s) \equiv \int_{0^-}^{\infty} v(t)e^{-st} dt$$

Other terms used are  $\delta(t)$ , which is the impulse (or delta) function, and  $u(t)$ , which is the unit step function.

$$\delta(t) \equiv \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases} \quad \text{with} \quad \int_{-\infty}^{\infty} \delta(t)dt = 1, \quad u(t) \equiv \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

LIST:	Name	t-domain	s-domain	Condition
	definition	$v(t)$	$V(s)$	
	linearity	$av_1(t) + bv_2(t)$	$aV_1(s) + bV_2(s)$	
	delay	$v(t - a)u(t - a)$	$e^{-as}V(s)$	$a \geq 0$
	multiply by $t$	$tv(t)$	$-\frac{d}{ds}V(s)$	
	multiply by $e^{-at}$	$e^{-at}v(t)$	$V(s) _{s+a}$ replaces $s$	$a \geq 0$
	divide by $t$	$\frac{v(t)}{t}$	$\int_s^{\infty} V(\underline{s})d\underline{s}$	
	derivative	$\frac{d}{dt}v(t)$	$sV(s) - v(t=0^-)$	
	nth derivative	$\frac{d^n}{dt^n}v(t)$	$s^nV(s) - s^{n-1}v(t) _{t=0^-}$ $-s^{n-2}\frac{d}{dt}v(t) _{t=0^-}$ $\vdots$ $-s^0\frac{d^{n-1}}{dt^{n-1}}v(t) _{t=0^-}$	
	integral	$\int_{0^-}^t v(t)dt$	$\frac{V(s)}{s}$	
	time scaling	$v(at)$	$\frac{1}{a}V(s) _{\frac{s}{a}}$ replaces $s$	$a \geq 0$

**REF:** [1] James A. Nilsson, Susan A. Riedel, *Electric Circuits*, 8th Ed., Upper Saddle River, NJ: Prentice Hall, 2007.