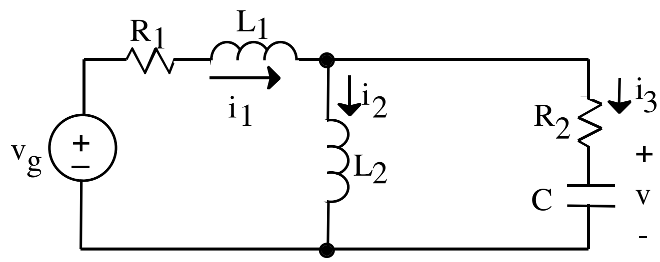
2. (25 points)



At t = 0, $v_g(t)$ switches instantaneously from $-v_o$ to $+v_o$.

a. Write the state-variable equations in terms of the state vector

$$x = \begin{bmatrix} i_1 \\ i_2 \\ v \end{bmatrix}$$

b. Evaluate the state vector x at $t = 0^+$.

ans: a)

$$\frac{di_1}{dt} = -\frac{(R_1 + R_2)}{L_1} i_1 + \frac{R_2}{L_1} i_2 - \frac{1}{L_1} v + \frac{1}{L_1} v_o$$

$$\frac{di_2}{dt} = \frac{R_2}{L_2} i_1 - \frac{R_2}{L_2} i_2 + \frac{1}{L_2} v$$

$$\frac{dv}{dt} = \frac{1}{C} i_1 - \frac{1}{C} i_2$$

b)

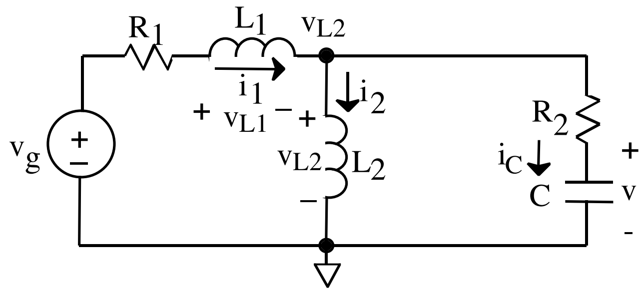
$$\begin{bmatrix} i_1 \\ i_2 \\ v \end{bmatrix}_{t=0^+} = \begin{bmatrix} -v_o / R_1 \\ -v_o / R_1 \\ 0\ V \end{bmatrix}$$

sol'n: (a) Our Equations must have the derivative of a state variable on the left and only state variables and constants on the right.

Find an equation for di_1/dt from $v_{L1} = L_1\, di_1/dt$ or $di_1/dt = v_{L1}/L_1$. Similarly, $di_2/dt = v_{L2}/L_2$.

We find an equation for dv/dt from $i_C = C$ dv/dt.

The diagram below shows the polarities of v_{L1}, v_{L2}, and i_C. It also shows a reference and node voltage.



We treat L_1 and L_2 as current sources and v as a voltage source, and we solve for v_{L1}, v_{L2}, and i_C using standard techniques (such as node-voltage or superposition). Each variable we are solving for is the voltage for a current source or the current for a voltage source. Thus, they are variables we solve for indirectly: we solve the circuit and then use Kirchhoff's laws to find the value we are looking for in terms of other values.

Since the current through R_2 is $i_1 - i_2$, we may solve for node voltage, v_{L2}, directly in the above circuit: (We could also use the node-voltage method.)

$$v_{L2} = v + (i_1 - i_2)R_2$$

This happens to be one of the variables we wish to solve for, and we find the equation for di_2/dt by dividing the equation by L_2.

$$\frac{di_2}{dt} = \frac{v_{L2}}{L_2} = \frac{1}{L_2}\left[v + (i_1 - i_2)R_2\right]$$

or

$$\frac{di_2}{dt} = \frac{v_{L2}}{L_2} = \frac{R_2}{L_2}i_1 - \frac{R_2}{L_2}i_2 + \frac{1}{L_2}v$$

We derive v_{L1} from a voltage loop on the left.

$$v_{L1} = v_g - i_1R_1 - (i_1 - i_2)R_2 - v$$

We find the equation for di_1/dt by dividing the equation by L_1 and substituting v_o for v_g.

$$\frac{di_1}{dt} = \frac{v_{L1}}{L_1} = \frac{1}{L_1}\left[v_g - i_1R_1 - (i_1 - i_2)R_2 - v\right]$$

or

$$\frac{di_1}{dt} = \frac{v_{L1}}{L_1} = -\frac{R_1 + R_2}{L_1} i_1 + \frac{R_2}{L_1} i_2 - \frac{1}{L_1} v + \frac{1}{L_1} v_o$$

We derive i_C from a current summation at the top node.

$$i_C = i_1 - i_2$$

We find the equation for dv/dt by dividing the equation by C.

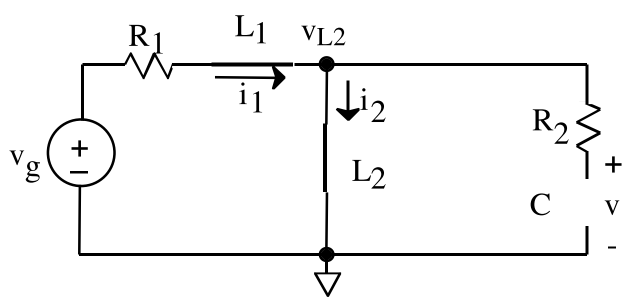
$$\frac{dv}{dt} = \frac{i_C}{C} = \frac{1}{C}(i_1 - i_2)$$

or

$$\frac{dv}{dt} = \frac{i_C}{C} = \frac{1}{C} i_1 - \frac{1}{C} i_2$$

sol'n: (b) We find initial conditions by looking at the circuit at time $t = 0^-$. Because we are dealing with state variables, (i.e., energy variables), their values cannot change instantly and will be the same at time $t = 0^-$ and $t = 0^+$.

At time $t = 0^-$, the circuit has reached equilibrium; voltages and currents are no longer changing; derivatives are zero; $v_L = L\, di_L/dt = 0$ and $i_C = dv_C/dt = 0$. Thus, the inductors look like wires and the capacitor looks like an open circuit.



Since no current flows through R_2, we may find i_1 (which equals i_2) from the loop on the left side.

$$i_1(0^-) = i_2(0^-) = \frac{v_g}{R_1} = \frac{-v_o}{R_1}$$

Since no current flows through R_2, there is no voltage drop across R_2. Thus, v equals the voltage drop across the wire representing L_2. In other words, we have

$$v(0^-) = 0$$

The state variables have the same value at time $t = 0^+$ as at $t = 0^-$.