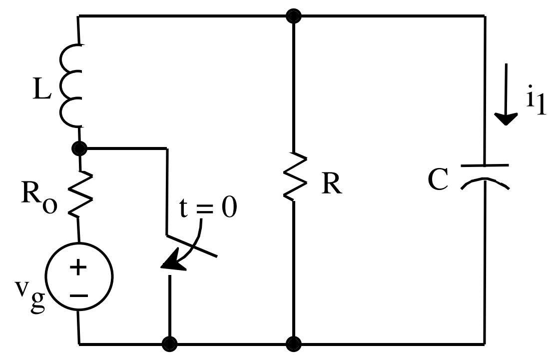
1. (25 points)



v_g is a dc voltage source
After having been open for a long time, the switch is closed at t = 0.

a. Give expressions for $i_1(0+)$ and $i_1'(0+)$, (i.e., di_1/dt at $t = 0^+$), in terms of no more than v_g, R_o, R, L, and C.

b. For L = 10 μH, choose R and C so that the system is underdamped and $\alpha = 3 \cdot 10^6$ rad/s, $\omega_d = 4 \cdot 10^6$ rad/s.

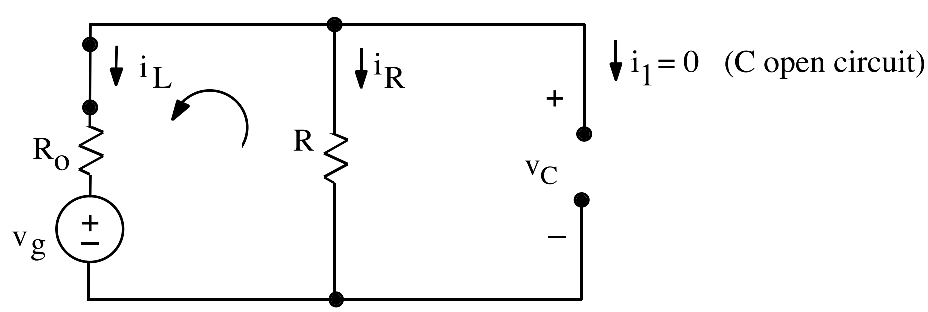
ans: a) $i_1(0^+) = 0A$

$$\left. \frac{di_1(t)}{dt} \right|_{t=0^+} = -\frac{v_g}{L} \frac{R}{R + R_o}$$

b) C = 4nF, R = 41.7Ω

sol'n: (a) To determine what the L and C are doing at $t = 0^+$, we find the value of $i_L(t = 0^-)$ and $v_C(t = 0^-)$ and argue that these values will be the same at time $t = 0^+$, since they cannot change instantly.

We find $i_L(t = 0^-)$ and $v_C(t = 0^-)$ from circuit model for $t = 0^-$: L acts like wire, C acts like open.

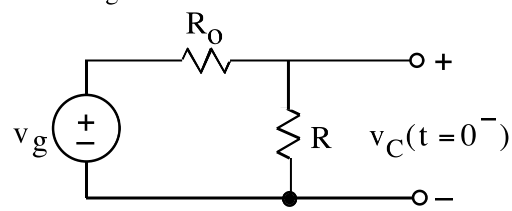


Current circulates only in the left inner loop, meaning $i_L = -i_R$. The current in this loop is just the source voltage divided by the sum of R and R_o.

$$i_L(t = 0^-) = -\frac{v_g}{R + R_o}$$

The right side looks like two dangling wires carrying no current but measuring the voltage drop, v_C, across R. The voltage drop is given by the voltage divider formula for R and R_o across v_g.

$$v_C(t = 0^-) = v_g \frac{R}{R + R_o}$$ circuit is

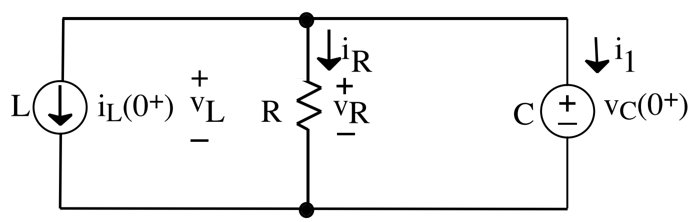


Since i_L and v_C cannot change instantly, we have

$$i_L(t = 0^+) = i_L(t = 0^-) = -\frac{v_g}{R + R_o},$$

$$v_C(t = 0^+) = v_C(t = 0^-) = v_g \frac{R}{R + R_o}.$$

The instant after the switch is closed, we model the inductor as a current source and the capacitor as a voltage source:



Henceforth, we work towards final answers involving only the state variables i_L and v_C. Since these are the only sources in the circuit, our answers will be in terms of state variables if we solve the circuit using standard techniques such as node-voltage.

If we place a reference on the bottom center node, then the top center node voltage is $v_C(0^+)$, owing to the voltage source on the right. Since i_1 is a current flowing through a voltage source, we find its value by summing currents out of the top-center node.

$$i_L + i_R + i_1 = 0A$$

$$i_1(0^+) = -i_L(0^+) - \frac{v_C(0^+)}{R} = \frac{v_g}{R + R_o} - \frac{v_g}{R + R_o} = 0A$$

This answer seems suspicious because the current before $t = 0^+$ was zero, too, and something has to change... The answer is correct, however, because the inductor and resistor currents have not changed. They are both determined by state variables that have not changed.

Qualitatively, what happens in this circuit is that i_L cannot change instantly when the switch closes. Before the switch closes, R carries all of i_L (because C acts like open circuit). After the switch closes, i_R cannot change instantly (because v_C is across R and v_C cannot change instantly). Thus, R still carries i_L just after the switch closes. This implies that $i_1(0^+) = 0$.

To find the derivative of i_1 at $t = 0^+$, we differentiate the equation for i_1 in terms of state variables found above and plug in $t = 0^+$ ***after*** we differentiate. (Note that, if we differentiate $i_L(0^+)$, then we are differentiating a constant and we get zero.)

$$i_1(t) = -i_L(t) - \frac{v_C(t)}{R}$$

$$\frac{di_1(t)}{dt} = -\frac{di_L(t)}{dt} - \frac{1}{R}\frac{dv_C(t)}{dt}$$

We convert these derivatives of state variables back into variables that are not derivatives by rearranging basic component equations for L and C:

$$v_L(t) = L\frac{di_L(t)}{dt} \Rightarrow \frac{di_L(t)}{dt} = \frac{v_L(t)}{L}$$

$$i_C(t) = C\frac{dv_C(t)}{dt} \Rightarrow \frac{dv_C(t)}{dt} = \frac{i_C(t)}{C}$$

We have

$$\left.\frac{di_1(t)}{dt}\right|_{t=0^+} = -\left.\frac{di_L(t)}{dt}\right|_{t=0^+} - \left.\frac{1}{R}\frac{dv_C(t)}{dt}\right|_{t=0^+}$$

$$\left.\frac{di_1(t)}{dt}\right|_{t=0^+} = -\left.\frac{v_L(t)}{L}\right|_{t=0^+} - \left.\frac{1}{R}\frac{i_C(t)}{C}\right|_{t=0^+}$$

$$\left.\frac{di_1(t)}{dt}\right|_{t=0^+} = -\frac{v_L(0^+)}{L} - \frac{1}{R}\frac{i_C(0^+)}{C}$$

Returning to the circuit diagram for $t = 0^+$, we solve for $v_L(0^+)$ and $i_C(0^+)$ using standard methods. In this case, we have $v_L(0^+) = v_C(0^+)$ and $i_C(0^+) = i_1(0^+)$. Plugging in these known values, we have

$$\left.\frac{di_1(t)}{dt}\right|_{t=0^+} = -\frac{v_C(0^+)}{L} - \frac{1}{R}\frac{i_1(0^+)}{C} = -\frac{v_g}{L}\frac{R}{R+R_o} - \frac{1}{R}\frac{0A}{C}$$

$$\left.\frac{di_1(t)}{dt}\right|_{t=0^+} = -\frac{v_g}{L}\frac{R}{R+R_o}.$$

sol'n: **(b)** $\alpha = 3$ M/s = 1/(2RC) for parallel RLC

$\omega_d = 4\ M/s = \sqrt{\omega_o^2 - \alpha^2}$ underdamped RLC

Thus, $\omega_d^2 = 4^2\ M^2/s^2 = \omega_o^2 - 3^2\ M^2/s^2$.

From $3^2 + 4^2 = 5^2$ Pythagorean triple we have

$$\omega_o^2 = 5^2 M^2/s^2$$

$$\omega_o^2 = \frac{1}{LC} \Rightarrow C = \frac{1}{\omega_o^2 L} = \frac{1}{5^2 M^2 10\mu} F = \frac{1}{250\ M} F$$

$$C = \frac{1}{250}\ \mu F = \frac{1k}{250}\ nF = 4nF.$$

Now use the definition of alpha

$$\alpha = \frac{1}{2RC} \Rightarrow \qquad R = \frac{1}{2\alpha C} = \frac{1}{2(3)M}\frac{\Omega}{4n} = \frac{1\,k\Omega}{24} = 41.7\ \Omega.$$