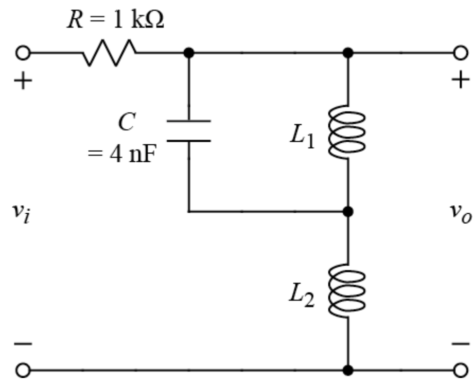
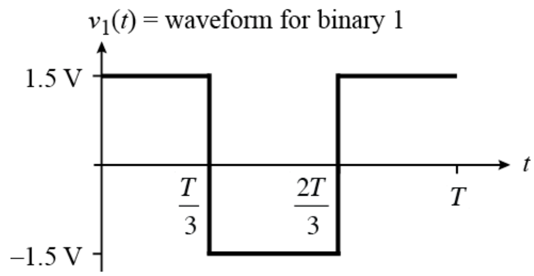
Ex:





T = one period of $v_i(t) = 2\pi$ ns

$$v_1(t) = \begin{cases} 1.5\text{ V} & 0 \le t < T/3 \\ -1.5\text{ V} & T/3 \le t < 2T/3 \\ 1.5\text{ V} & 2T/3 \le t \le T \end{cases}$$

The above filter circuit is being considered for use in a communication system to detect whether received signals represent binary zeros or binary ones. The plan is to use an inexpensive design with rectangular waveforms (rather than sinusoids). A zero will be signaled by a square wave (not shown), and a one will be signaled by a rectangular wave having 2/3 duty cycle (shown above). The filter for detecting a zero is designed to pass the fundamental frequency of the waveforms, which is the same as the fundamental frequency of the waveform shown above. The issues addressed in this problem are the design of the filter and how well it blocks the waveform representing a "one".

a) Find values of $L_1 \neq 0$ and $L_2 \neq 0$ such that the magnitude of the filter's transfer function, H, equals one for the fundamental frequency, ω_0, and zero for frequency $3\omega_0/2$, (which the engineer proposing the circuit believes is present in the signal for a "one").

b) Find the numerical value of coefficient a_v, (the DC input to the filter), of the signal into the filter for the Fourier series for $v_1(t)$ in problem 3:

$$v_1(t) = a_v + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)$$

c) Find the numerical value of the magnitude, $\sqrt{a_1^2 + b_1^2}$, of the fundamental-frequency of the signal into the filter in problem 3.

sol'n a) We can achieve a gain (i.e., $|H| = 1$) by having a resonance of $L_1 \| C$ (i.e. having $j\omega L \| \frac{1}{j\omega C} = \infty$) at $\omega = \omega_0$. At resonance,

we have $j\omega L + \frac{1}{j\omega C} = 0$:

$$j\omega_0 L \| \frac{1}{j\omega_0 C} = \frac{j\omega_0 L / j\omega_0 C}{j\omega_0 L + 1/j\omega_0 C} = \frac{L/C}{0} = \infty$$

This gives $|H| = \dfrac{j\omega_0 L \| \frac{1}{j\omega_0 C}}{j\omega_0 L \| \frac{1}{j\omega_0 C} + R} = \frac{\infty}{\infty} = 1.$

Since $\omega_0^2 = \frac{1}{L_1 C} = \left(\frac{2\pi}{T}\right)^2 = \left(\frac{2\pi}{2\pi n}\right)^2 = 1\,G^2 r^2/s^2$,

we can solve for L_1:

$$L_1 = \frac{1}{\omega_0^2 C} = \frac{1}{1G^2\, 4n} H = 250\, pH$$

To achieve $|H| = 0$, we create a resonance of $j\omega L_1 \| \frac{1}{j\omega C}$ and $j\omega L_2$ so that C, L_1, and L_2 act like a wire: at $\frac{3\omega_0}{2}$

$$\frac{j\frac{3}{2}\omega_0 L_1 \cdot \frac{1}{j\frac{3\omega_0}{2} C}}{j\frac{3}{2}\omega_0 L_1 + \frac{1}{j\frac{3}{2}\omega_0 C}} + j2\omega_0 L_2 = 0$$

$$\frac{\cancel{j1.5G} \cdot 250p \cdot 1 / \cancel{j1.5G}\, 4n\,\Omega}{j1.5G \cdot 250p + 1/j1.5\cancel{G}\, 4\cancel{n}} + j\,2G\,L_2 = 0$$

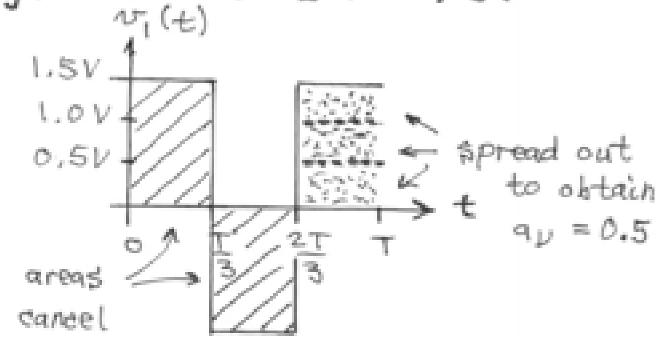
$$\frac{1/16}{j\frac{1.5}{4} - j\frac{1}{6}} + j1.5G L_2 = 0$$

$$\frac{3}{j18 - j8} + j1.5GL_2 = 0$$

$$1.5GL_2 = \frac{3}{10}$$

$$L_2 = \frac{3}{15G} = 200 \text{ pF}$$

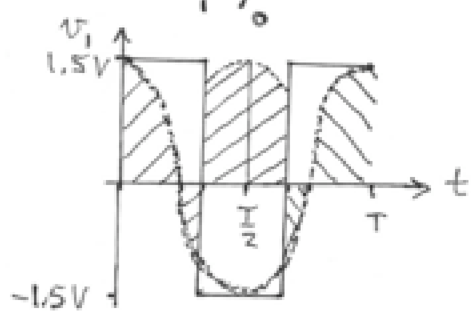
b) The first and second sections of $v_1(t)$ have equal but opposite areas that cancel out. If we spread out the area under the third section over the entire interval from 0 to T, we have an average height of $0.5V = 1.5V/3$:



$$a_v = \frac{1}{2}V$$

To calculate a_1 and b_1, we use pictures to help evaluate the integrals:

$$a_1 = \frac{2}{T}\int_0^T v_1(t)\cos(\omega_0 t)\,dt$$



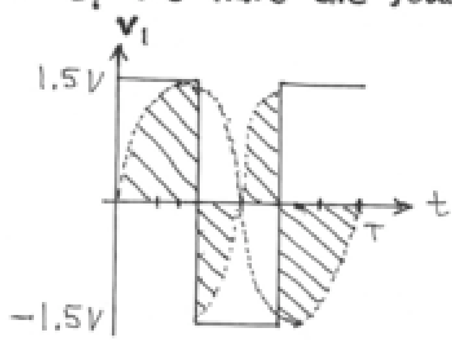
The area under the curve (i.e., the integral) is equal to twice the area under the curve from 0 to $\frac{T}{2}$. We break the integral into two pieces.

$$a_1 = 2\left(\frac{2}{T}\right)\left[\int_0^{T/3} 1.5\cos(\omega_0 t)\,dt + \int_{T/3}^{T/2} -1.5\cos(\omega_0 t)\,dt\right]$$

$$= \frac{4}{T}\left[\left.\frac{1.5\sin\left(\frac{2\pi}{T}t\right)}{\frac{2\pi}{T}}\right|_0^{T/3} + \left.\frac{-1.5\sin\left(\frac{2\pi}{T}t\right)}{\frac{2\pi}{T}}\right|_{T/3}^{T/2}\right]$$

$$= \frac{4}{2\pi}(1.5)\left[\frac{\sqrt{3}}{2} - -\frac{\sqrt{3}}{2}\right] = \frac{3\sqrt{3}}{\pi}$$

For b_1 we have the following picture:



We see that the areas cancel out, so

$b_1 = 0$, (which also follows from $v_1(t)$ being an even func).

Thus, $\sqrt{a_1^2 + b_1^2} = a_1 = 3\sqrt{3}/\pi$.