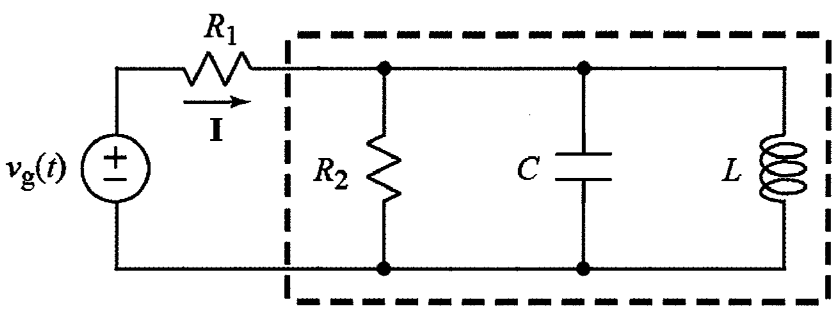
HOMEWORK #7 prob 2 solution

Ex:



$v_g(t) = 5\cos(2\text{M}t)\text{V}$

$R_1 = 0.1\ \Omega$ $\quad$ $R_2 = 1\ \Omega$

$C = 1\mu\text{F}$ $\quad$ $L = 125\ \text{nH}$

a) Calculate **I**.

b) Calculate the complex power, S, for the components inside the box.

soln: a) We use phasor currents and voltages, and impedances.

$$V_g = P[v_g(t) = 5\cos(2\text{M}t)\ V]$$

$$V_g = 5\angle 0°\ V$$

$$\frac{1}{j\omega C} = \frac{1}{j2\text{M}(1\mu)}\ \Omega = -j\frac{1}{2}\ \Omega$$

$$j\omega L = j\,2\text{M}(125\text{n})\ \Omega = j\frac{1}{4}\ \Omega$$

Now we calculate z_{box}, starting with the reactive components, L and C. (The idea is to keep imaginary quantities together and real quantities together.)

$$\frac{1}{j\omega C} \parallel j\omega L = \frac{1}{\frac{1}{\frac{1}{j\omega C}} + \frac{1}{j\omega L}} = \frac{1}{j\omega C + \frac{-j}{\omega L}}$$

$$\frac{1}{j\omega C} \,\|\, j\omega L = \frac{1}{j2 - j4\,\Omega} = \frac{1}{-j2\,\Omega}$$

$$= j\frac{1}{2}\,\Omega$$

$$z_{box} = 1\,\Omega \,\|\, j\frac{1}{2}\,\Omega = \frac{1}{\frac{1}{1\,\Omega} + \frac{1}{j\frac{1}{2}}}$$

$$= \frac{1}{1-j2}\,\Omega \cdot \frac{1+j2}{1+j2}$$

$$= \frac{1+j2}{5}\,\Omega = 0.2 + j0.4\,\Omega$$

The current is equal to V_g divided by the total impedance in the circuit.

$$I = \frac{V_g}{0.1\,\Omega + 0.2 + j0.4\,\Omega} = \frac{5\angle 0°\,V}{0.3 + j0.4\,\Omega}$$

$$= \frac{5\angle 0°\,V}{0.5\angle 53°\,\Omega}$$

$$I = 10\angle -53°\ A$$

b) We have various formlae for S, based on a basic eq'n and Ohm's law, $V = IZ$:

$$S = \frac{VI^*}{2} = \frac{|I|^2}{2}Z = \frac{|V|^2}{2Z^*}$$

where $*$ means complex conjugate.

Here, we use $S = \frac{|I|^2}{2} Z_{box}$.

$$S = \frac{10^2}{2}\cdot(0.2 + j0.4)\ VA = 10 + j20\ VA$$