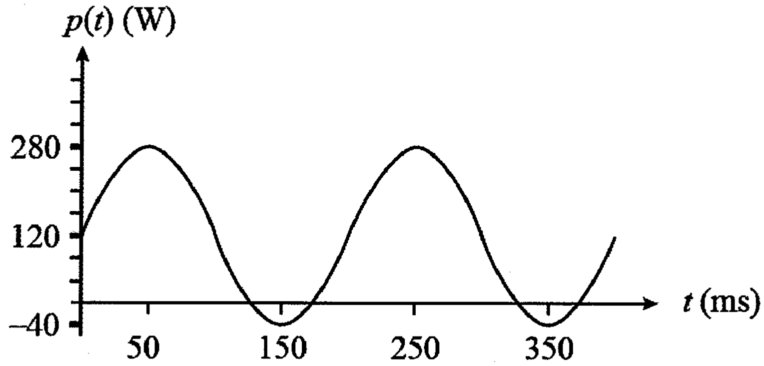
Ex:



Use the waveform of power versus time shown above to answer the following questions:

a) Determine the value of ω.

b) Calculate complex power $S = P + jQ$.

c) Can the phasor values of **V** and **I** be determined uniquely from the waveform? If so, find **V** and **I**.

sol'n: a) The power waveform as shown isn't necessarily phase-shifted so that the phase-shift of the current, $i(t)$, is zero. (In fact, the phase shift of the current cannot be zero, as will be seen later on.)

Consider the power waveform when the angle of the current is not zero:

$$p(t) = v(t)\, i(t)$$

$$'' = v_m \cos(\omega t + \theta_v)\, i_m \cos(\omega t + \theta_i)$$

$$p(t) = \frac{v_m i_m}{2} \cos(2\omega t + \theta_v + \theta_i) \quad \text{AC pwr}$$

$$+ \frac{v_m i_m}{2} \cos(\theta_v - \theta_i) \quad \text{DC pwr}$$

We compare this to the $p(t)$, which we will call $p_1(t)$, that we obtain when we assume $\theta_i = 0$:

$$p_1(t) = \frac{v_m i_m}{2} \cos(2\omega t + \theta_{v-i}) \qquad \text{AC pwr}$$
$$+ \frac{v_m i_m}{2} \cos(\theta_{v-i}) \qquad \text{DC pwr}$$

The key difference is that $p_1(t)$ has angle $\theta_v - \theta_i$, whereas $p(t)$ has angle $\theta_v + \theta_i$.

This discrepancy involves only the phase shift. The frequency of $p(t)$ is still 2ω.

We now find ω.

The period of $p(t)$ is $T = 250\text{ms} - 50\text{ms}$ or $T = 200$ ms.

So $\quad 2\omega = \frac{2\pi}{T} \quad$ or $\quad \omega = \frac{\pi}{T} = \frac{\pi}{200\text{ms}}$

or $\quad \omega = 5\pi$ rad/s

b) The complex power, S, is the phasor of the AC part of $p_1(t)$, (not the phasor of the AC part of $p(t)$).

We have only $p(t)$ not $p_1(t)$, but the DC offset of $p(t)$ and $p_1(t)$ is the same:

$$P = \frac{v_m i_m}{2} \cos(\theta_v - \theta_i) = 120 \text{ W}$$

(from graph of $p(t)$)

The magnitude of the AC pwr part of $p(t)$ and $p_1(t)$ is also the same, and the magnitude of the AC pwr is $|\tilde{S}| = \sqrt{P^2 + Q^2}$.

From the graph, the magnitude of the sinusoidal part of $p_1(t)$ is

$$|\tilde{S}| = 280W - 120W = 160\,W.$$

Now we can solve for Q.

$$160\,W = \sqrt{(120W)^2 + Q^2}$$

or

$$160^2 W^2 = 120^2 W^2 + Q^2$$

or

$$256(100)W^2 - 144(100)W^2 + Q^2$$

or

$$112(100)W^2 = Q^2$$

or

$$Q = \pm\sqrt{112} \cdot 10\text{ VAR}$$

or

$$Q = \pm 105.8\text{ VAR}$$

Note: we write Watts in different ways so we can tell which pwr quantity we are looking at, if all we see is a number. We use the following units:

P (W), Q (VAR), $\tilde{S}$ (VA)

VAR stands for Volt-amps reactive

c) Now we try to determine $\mathbf{V}$ and $\mathbf{I}$.

We observe that $|S| = \frac{v_m i_m}{2} = 160\,W$

and $P = \frac{v_M i_M}{2} \cos(\theta_{v-i}) = 120\,W.$

So $\frac{P}{Q} = \cos(\theta_{v-i})$

and

$$\theta_{v-i} = \cos^{-1}\left(\frac{P}{Q}\right) = \cos^{-1}\left(\frac{3}{4}\right)$$

$$\theta_{v-i} = 41.4^\circ = \theta_v - \theta_i$$

If we look at $p(t)$ at time $t=0$, we have

$$\begin{aligned} p(0) &= \frac{v_M i_m}{2} \cos(\theta_v + \theta_i) \\ &\quad + \frac{v_m i_m}{2} \cos(\theta_v - \theta_i) \\ &= \frac{v_m i_m}{2} \cos(\theta_v + \theta_i) \\ &\quad + P \\ &= P = 120\,W \text{ (from graph)} \end{aligned}$$

So $\frac{v_M i_m}{2} \cos(\theta_v + \theta_i) = 0$

This means $\theta_v + \theta_i = \pm 90^\circ$. Looking at $p(t)$, we see a waveform that looks like $\sin(2\omega t)$ rather than $-\sin(2\omega t)$ starting from $t=0$.

So $\theta_v + \theta_i = -90^\circ$

and $\theta_v - \theta_i = 41.4^\circ$.

Summing eq'ns, we have

$$2\theta_v = -90° + 41.4° = -48.6°$$

or

$$\theta_v = -24.3°$$

It follows that $\theta_i = -65.7°$.

We have $|S| = \frac{v_m i_m}{2} = 160\,W$

or $\quad v_m i_m = 320\ W.$

Nowhere in our eq'ns do we see anything but $v_m i_m$. Thus, we have no way of uniquely determining v_m and i_m.

$$\mathbf{V} = v_m \angle -24.3°$$

$$\mathbf{I} = i_m \angle -65.7°$$

and $\quad v_m i_m = 320\ W$

Note: $\frac{v_m}{i_m} = |z|$, so we could find v_m and i_m if we knew z.