Ex: Find the inverse Laplace transform of $\frac{24s}{(s+5)^4}$. Note: $\mathcal{L}\{t^n e^{-at}\} = \frac{n!}{(s+a)^{n+1}}$

SOL'N: One approach is to use the identity for d/dt:

$$\mathcal{L}\left\{\frac{d}{dt}v(t)\right\} = sV(s) - v(0^-)$$

(We hope $v(0^-) = 0$.)

So $V(s) = \frac{24}{(s+5)^4} = 4 \cdot \frac{3!}{(s+5)^4}$

$$\mathcal{L}^{-1}\{V(s)\} = 4\mathcal{L}^{-1}\left\{\frac{3!}{(s+5)^4}\right\}$$

$$= 4t^3e^{-5t} \equiv v(t)$$

Fortunately, $v(0^-) = 0$.

$$\mathcal{L}^{-1}\{sV(s)\} = \frac{d}{dt}\left(4t^3e^{-5t}\right)$$

$$= 4\left(3t^2e^{-5t} + t^3(-5)e^{-5t}\right)$$

$$= \left(-20t^3 + 12t^2\right)e^{-5t}$$

or

$$\mathcal{L}^{-1}\left\{\frac{24s}{(s+5)^4}\right\} = \left(-20t^3 + 12t^2\right)e^{-5t}u(t)$$

Note: We multiply by $u(t)$ to remind ourselves that we don't know the value of the function for $t<0$.

Another approach is to use partial fractions.

$$\frac{24s}{(s+5)^4} = \frac{A}{(s+5)^4} + \frac{B}{(s+5)^3} + \frac{C}{(s+5)^2} + \frac{D}{s+5}$$

or

$$\frac{24s}{(s+5)^4} = \frac{A + B(s+5) + C(s+5)^2 + D(s+5)^3}{(s+5)^4}$$

Thus, $24s = A + B(s+5) + C(s+5)^2 + D(s+5)^3.$

$A = 24s\Big|_{s=-5}$ since right side becomes $A + B\cdot 0 + C\cdot 0^2 + D\cdot 0^3$

$A = -120$

Now we use derivatives to find B, C, and D.

$$\frac{d}{ds}(24s) = 24 = B + 2C(s+5) + 3D(s+5)^2$$

$$\frac{d}{ds}(24s)\Big|_{s=-5} = 24 = B + 2C\cdot 0 + 3D\cdot 0^2 = B$$

$B = 24$

$$\frac{d^2}{ds^2}(24s)\Big|_{s=-5} = \frac{d}{ds}(24)\Big|_{s=-5} = 0 = 2C + 6D(s+5)\Big|_{s=-5} = 2C$$

$C = 0$

$$\frac{d^3}{ds^3}(24s)\Big|_{s=-5} = 0 = 6D\Big|_{s=-5} = 6D$$

$D = 0$

Thus, $\dfrac{24s}{(s+5)^4} = \dfrac{-120}{(s+5)^4} + \dfrac{24}{(s+5)^3}$

$\quad$ " $= -20\cdot\dfrac{3!}{(s+5)^4} + 12\,\dfrac{2!}{(s+5)^3}$

So $\mathcal{L}^{-1}\left\{\frac{24s}{(s+5)^4}\right\} = \left[-20t^3e^{-5t} + 12t^2e^{-5t}\right]u(t)$

This is the same answer as before.