Ex: Find the inverse Laplace transform for the following expression:

$$F(s) = \frac{15s^2 + 186s + 624}{s^3 + 18s^2 + 112s + 160}$$

SOL'N: Using whatever method is convenient, we factor the denominator as follows:

$$s^3 + 18s^2 + 112s + 160 = (s+2)(s+8+j4)(s+8-j4)$$

We use a quadratic form for the complex roots.

$$(s+2)[(s+8)^2 + 4^2]$$

$$F(s) = \frac{A}{s+2} + \frac{Bs + C}{(s+8)^2 + 4^2}$$

Use a common denominator

$$F(s) = \frac{A[(s+8)^2+4^2] + (Bs+C)(s+2)}{(s+2)[(s+8)^2+4^2]}$$

and " $= \dfrac{15s^2 + 186s + 624}{(s+2)[(s+8)^2+4^2]}$

Equate numerators. Evaluate at convenient values of s.

$$A[(s+8)^2+4^2] + (Bs+C)(s+2)\Big|_{s=-2} = A(6^2+4^2)$$

$$15s^2 + 186s + 624\Big|_{s=-2} = 15(-2)^2 + 186(-2) + 624$$

or $52A = 312$

$A = 6$

Substitute for A and equate numerators:

$$6\left[(s+8)^2+4^2\right]+(Bs+C)(s+2)$$

$$= 15s^2+186s+624$$

For $s=0$, we have

$$6\left[8^2+4^2\right]+C(2) = 624$$

$$480+2C = 624$$

$$C = 72$$

For $s=-1$, we have

$$6\left[7^2+4^2\right]+(-B+72)\cdot 1 = 15-186+624$$

or $$6\left[65\right]+72-B = 453$$

$$B = +9$$

So $$F(s) = \frac{6}{s+2} + \frac{+9s+72}{(s+8)^2+4^2}$$

Now we write the last term as a decaying cos and sin:

$$\frac{+9s+72}{(s+8)^2+4^2} = \frac{C\,(s+\overset{\overset{a}{\shortparallel}}{8})}{(s+\underset{\underset{a}{\shortparallel}}{8})^2+\underset{\underset{\omega}{\shortparallel}}{4}^2} + \frac{D\cdot \overset{\overset{\omega}{\shortparallel}}{4}}{(s+\underset{\underset{a}{\shortparallel}}{8})^2+\underset{\underset{\omega}{\shortparallel}}{4}^2}$$

Numerators:

$$+9s+72 = C(s+8)+4D$$

$$+9s = Cs \Rightarrow C = +9$$

$$72 = 8C+4D \Rightarrow D = 0$$

$$F(s) = \frac{6}{s+2} + \frac{9(s+8)}{(s+8)^2+4^2}$$

$$\mathcal{L}^{-1}\{F(s)\} = \left[6e^{-2t} + 9e^{-8t}\cos(4t)\right]u(t)$$