Ex: Find the inverse Laplace transform for the following expression:

$$F(s)=\frac{-3s^2+99s-1200}{s^3+11s^2+100s+1100}$$

SOL'N: We first factor the denominator. Using a computer (e.g., Matlab®), we find the roots of the denominator are $s=-11$, $j10$, and $-j10$.

So $F(s)=\dfrac{-3s^2+99s-1200}{(s+11)(s+j10)(s-j10)}$

To avoid complex numbers we use the quadratic $(s+j10)(s-j10)=s^2+100$ in the denominator.

$$F(s)=\frac{-3s^2+99s-1200}{(s+11)(s^2+100)}$$

$$F(s)=\frac{A}{s+11}+\frac{Bs+C}{s^2+100}$$

$$F(s)=\frac{A(s^2+100)+(Bs+C)(s+11)}{(s+11)(s^2+100)}$$

So we must solve to make the numerator equal the original numerator.

$$A(s^2+100)+(Bs+C)(s+11)=-3s^2+99s-1200$$

We have $A(s^2+100)\Big|_{s=-11}=-3s^2+99s-1200\Big|_{s=-11}$

$$A\cdot 221=-363-1089-1200$$

$$A=-12$$

Now use convenient values of s to obtain eq'ns for B and C.

$$A(s^2+100)+(Bs+C)(s+11)\Big|_{s=0} = -3s^2+99s-1200\Big|_{s=0}$$

$$100\underset{-12}{\overset{}{A}} + 11C = -1200 \Rightarrow C=0$$

$$A(s^2+100)+(Bs+C)(s+11)\Big|_{s=1} = -3s^2+99s-1200\Big|_{s=1}$$

$$101\underset{}{\overset{-12}{A}} + (B+\overset{0}{C})\cdot 12 = -3+99-1200$$

$$-1212 + 12B = -1104$$

$$12B = 108$$

$$B = 9$$

$$F(s) = \frac{-12}{s+11} + \frac{9s}{s^2+100} \Leftarrow 9\cdot\frac{s}{s^2+\underset{\omega^2}{10^2}}$$

$$\mathcal{L}^{-1}\{F(s)\} = \left[-12e^{-11t} + 9\cos 10t\right]u(t)$$