Ex: Find the inverse Laplace transform for the following expression:

$$F(s) = \frac{7s+70}{s^2+8s+25}$$

Sol'n: The denominator factors as follows:

$$s^2+8s+25 = \left(s+\frac{8}{2}\right)^2 + 25 - \left(\frac{8}{2}\right)^2$$

$$= (s+4)^2 + 3^2$$

So $F(s) = \dfrac{7(s+10)}{(s+4)^2+3^2}$

We rewrite this as a decaying exponential times a cos plus a sine:

$$\frac{7(s+10)}{(s+4)^2+3^2} = 7\left[\frac{A\overset{a}{\overset{\shortparallel}{(s+4)}}}{(s+\underset{\underset{a}{\shortparallel}}{4})^2+\underset{\underset{\omega}{\shortparallel}}{3}^2} + \frac{B\cdot \overset{\omega}{\overset{\shortparallel}{3}}}{(s+\underset{\underset{a}{\shortparallel}}{4})^2+\underset{\underset{\omega}{\shortparallel}}{3}^2}\right]$$

So $A(s+4) + B(3) = s+10$

We match coeff's of powers of s:

$As = s$ and $A(4) + B(3) = 10$

$A = 1$ $\quad 4 + 3B = 10$

$B = 2$

$$F(s) = 7\frac{(s+4)}{(s+4)^2+3^2} + \frac{14\cdot 3}{(s+4)^2+3^2}$$

$$\mathcal{L}^{-1}\{F(s)\} = \left[7e^{-4t}\cos 3t + 14e^{-4t}\sin 3t\right]u(t)$$