EX: Find the Laplace transform of the following waveform:

$t\sin(\omega t)\cos(\omega t)$

SOL'N: We transform this function into a form that may be found in a table of Laplace transforms.

$$\sin(\omega t)\cos(\omega t) = \frac{1}{2}\sin(\omega t + \omega t)$$

$$= \frac{1}{2}\sin(2\omega t)$$

Thus, we are computing the following Laplace transform:

$$\mathcal{L}\{t\sin(\omega t)\cos(\omega t)\} = \mathcal{L}\{\frac{1}{2}t\sin(2\omega t)\}$$

Now we use the identity for multiplication by t:

$$\mathcal{L}\{t\,v(t)\} = -\frac{d}{ds}V(s)$$

We also use the transform pair for sin():

$$\mathcal{L}\{\sin(\omega t)\} = \frac{\omega}{s^2+\omega^2}$$

We have

$$\mathcal{L}\{\frac{1}{2}t\sin(2\omega t)\} = -\frac{d}{ds}\left(\frac{1}{2}\cdot\frac{2\omega}{s^2+(2\omega)^2}\right)$$

$$= -\omega\,\frac{-1\,(2s)}{[s^2+(2\omega)^2]^2}$$

$$= \frac{2\omega s}{[s^2+4\omega^2]^2}$$