EX: Find the Laplace transforms of the following waveform:

$$t\int_0^t te^{-at}\,dt$$

**SOL'N:** Use integration identity:

$$\mathcal{L}\left\{\int_0^t v(t)\,dt\right\} = \frac{1}{s}V(s)$$

so $$\mathcal{L}\left\{\int_0^t te^{-at}\,dt\right\} = \frac{1}{s}\mathcal{L}\{te^{-at}\}$$

Use mult. by $t$ identity:

$$\mathcal{L}\{tv(t)\} = -\frac{d}{ds}V(s)$$

so $$\mathcal{L}\{te^{-at}\} = -\frac{d}{ds}\mathcal{L}\{e^{-at}\} = -\frac{d}{ds}\frac{1}{s+a}$$

$$= \frac{+1}{(s+a)^2}$$

So $$\mathcal{L}\left\{\int_0^t te^{-at}\,dt\right\} = \frac{1}{s(s+a)^2}$$

Use mult by $t$ again:

$$\mathcal{L}\left\{t\int_0^t te^{-at}\,dt\right\} = -\frac{d}{ds}\frac{1}{s(s+a)^2}$$

$$= \frac{-1}{(s+a)^2} + \frac{+2s}{(s+a)^3}$$

$$= \frac{2s-(s+a)}{(s+a)^3}$$

$$= \frac{s-a}{(s+a)^3}$$