

EX: Find the Laplace transforms of the following waveform:

$$f(t) = t \cos(\omega t) e^{-at}$$

SOL'N: We could apply identities in several different ways. Here, the multiplication by t identity is used:

$$\mathcal{L}\{t v(t)\} = -\frac{d}{ds} V(s)$$

$$\text{so } v(t) = \cos(\omega t) e^{-at}$$

$$V(s) = \mathcal{L}\{e^{-at} \cos(\omega t)\} = \frac{s+a}{(s+a)^2 + \omega^2}$$

$$\mathcal{L}\{t v(t)\} = -\frac{d}{ds} \frac{s+a}{(s+a)^2 + \omega^2}$$

$$= \frac{-1}{(s+a)^2 + \omega^2} + \frac{(s+a) \cdot 2s}{[(s+a)^2 + \omega^2]^2}$$

$$= \frac{2s(s+a) - 1}{[(s+a)^2 + \omega^2]^2}$$

$$= \frac{2s^2 + 2sa - s^2 - 2as - a^2 - \omega^2}{[(s+a)^2 + \omega^2]^2}$$

$$\mathcal{L}\{t \cos(\omega t) e^{-at}\} = \frac{s^2 - (a^2 + \omega^2)}{[(s+a)^2 + \omega^2]^2}$$