

Ex: Find the Laplace transforms of the following waveform:

$$\frac{d}{dt} [t \sin(\omega t) + t^2]$$

SOL'N: Use derivative identity and mult by t identity:

$$\mathcal{L} \left\{ \frac{d}{dt} v(t) \right\} = s V(s) - v(0^-); \quad \mathcal{L} \left\{ t v(t) \right\} = -\frac{d}{ds} V(s)$$

$$\begin{aligned} \mathcal{L} \left\{ \frac{d}{dt} [t \sin(\omega t) + t^2] \right\} &= -\frac{d}{ds} \mathcal{L} \left\{ \sin(\omega t) \right\} - \bar{\omega} \sin(\omega \cdot 0^-) \\ &\quad + \mathcal{L} \left\{ t^2 \right\} \end{aligned}$$

$$= s \left(-\frac{d}{ds} \frac{\omega}{s^2 + \omega^2} \right) - 0 + \frac{2}{s^3}$$

$$\mathcal{L} \left\{ \frac{d}{dt} [t \sin(\omega t) + t^2] \right\} = \frac{2\omega s^2}{(s^2 + \omega^2)^2} + \frac{-\omega}{s^2 + \omega^2} + \frac{2}{s^3}$$