Ex: Find the Laplace transforms of the following waveform:

$(t-a)\cos(t-b)u(t-a)$ where $a>0$

SOL'N: Use delay identity:

$$\mathcal{L}\{v(t-a)u(t-a)\} = e^{-as}\mathcal{L}\{v(t)\}, \quad a>0$$

To recognize $(t-a)\cos(t-b)$ as a function of $t-a$, we substitute $t=(t-a)+a$ where necessary.

$$(t-a)\cos(t-b) = (t-a)\cos((t-a)+a-b)$$

or

$$v(t-a) = (t-a)\cos((t-a)+a-b)$$

We obtain $v(t)$ by substituting t for $t-a$.

$$v(t) = t\cos(t+a-b)$$

Now we want to find $\mathcal{L}\{v(t)\} \equiv V(s)$.

$$\cos(t+a-b) = \cos(t)\cos(a-b) - \sin(t)\sin(a-b)$$

We observe that $\cos(a-b)$ and $\sin(a-b)$ are constants.

So $\mathcal{L}\{\cos(t)\cos(a-b) - \sin(t)\sin(a-b)\}$

$$= \cos(a-b)\mathcal{L}\{\cos(t)\} - \sin(a-b)\mathcal{L}\{\sin(t)\}$$

$$= \cos(a-b)\frac{s}{s^2+1^2} - \sin(a-b)\frac{1}{s^2+1^2}$$

Finally, $\mathcal{L}\{(t-a)\cos(t-b)u(t-a)\} = e^{-as}\dfrac{\cos(a-b)s - \sin(a-b)}{s^2+1}$.