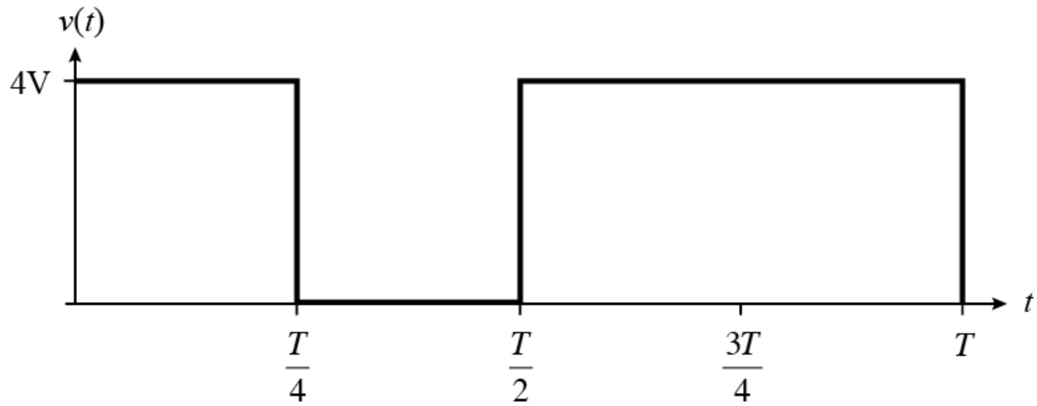
Ex:



One period, T, of a function $v(t)$ is shown above. The formula for $v(t)$ is

$$v(t) = \begin{cases} 4\text{ V} & 0 < t < T/4 \\ 0\text{V} & T/4 < t < T/2 \\ 4\text{ V} & T/2 < t < T \end{cases}$$

Find the numerical value of the following coefficients of the Fourier series for $v(t)$:

a) a_v b) a_2

c) b_2 d) b_4

SOL'N: a) a_ν = ave value of $v(t) = \frac{1}{T}\int_0^T v(t)\,dt$

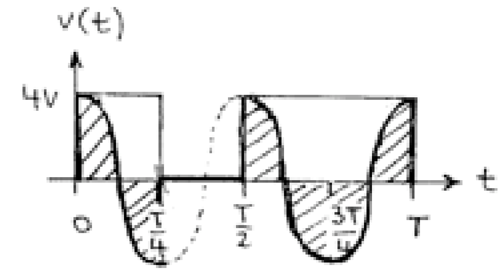
$= \frac{1}{T} \cdot$ area under $v(t)$

$= \frac{1}{T}\left(4V \cdot \frac{T}{4} + 4V \cdot \frac{T}{2}\right)$

$a_\nu = 3V$ (or imagine $v(t)$ spread out to a uniform height)

b) $a_2 = \frac{2}{T}\int_0^T v(t)\cos(2\omega_0 t)\,dt$ where $\omega_0 = \frac{2\pi}{T}$

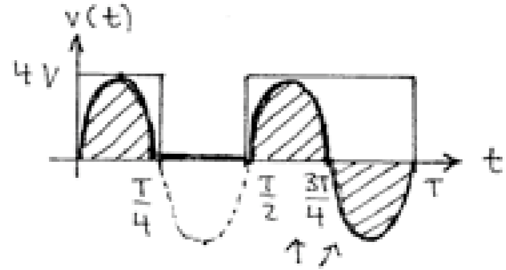
We draw a picture of $v(t)\cos(\omega_0 t)$ and see what the area under the curve, (i.e., the integral), is.



The areas all cancel out. $\therefore a_2 = 0 \text{ V}$

c) $b_2 = \frac{2}{T}\int_0^T v(t)\sin(2\omega_0 t)\,dt$, $\omega_0 = \frac{2\pi}{T}$

We draw a picture of $v(t)\sin(2\omega_0 t)$ and see what the area under the curve is.



Areas cancel out

We only need the first area:

$$b_2 = \frac{2}{T}\int_0^{T/4} 4V \cdot \sin(2\omega_0 t)\,dt$$

$$= \frac{2}{T}\cdot 4V\left(\frac{-\cos(2\omega_0 t)}{2\omega_0}\right)\Bigg|_0^{T/4}, \quad \omega_0 = \frac{2\pi}{T}$$

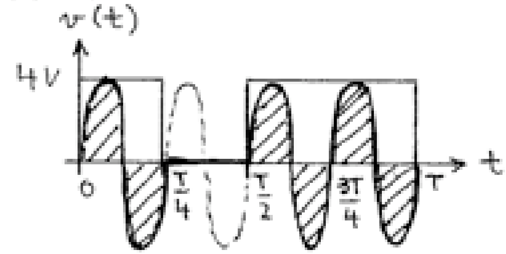
$$= \frac{\cancel{2}}{\cancel{T}}\cdot \overset{2}{\cancel{4}}V\left(\frac{-\cos\left(2\cdot\frac{2\pi}{T}t\right)}{\cancel{2}\cdot\frac{\cancel{2}\pi}{\cancel{T}}}\right)\Bigg|_0^{T/4}$$

$$= \frac{2}{\pi}V\left(-\cos\pi - -\cos 0\right)$$

$$b_2 = \frac{4}{\pi}V$$

d) $b_4 = \frac{2}{T}\int_0^T v(t)\sin(4\omega_0 t)\,dt$

picture:



All the areas cancel out.

$\therefore b_4 = 0V$