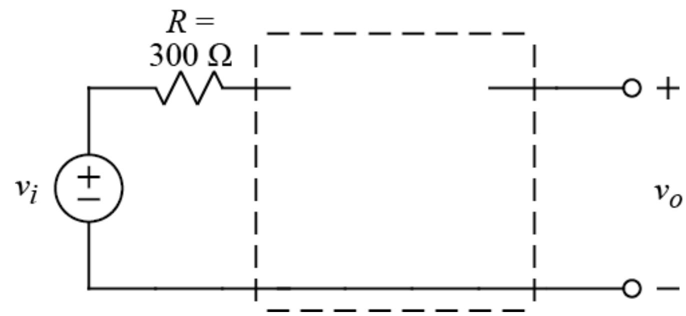
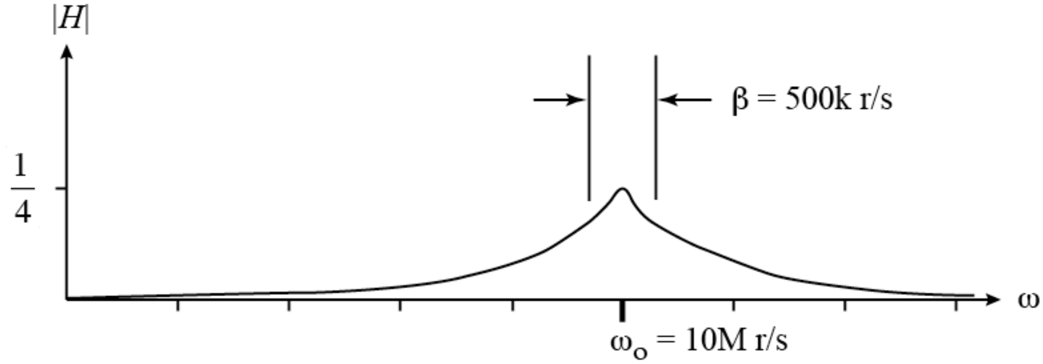
HOMEWORK #4 prob 1 solution

Ex:





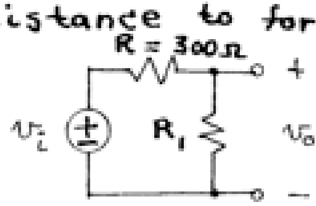
Given the resistor connected as shown and using not more than one each R, L, and C in the dashed-line box, design a circuit to go in the dashed-line box that will produce the **band-pass** $|H(j\omega)|$ vs. ω shown above. That is:

$$\max_{\omega} |H(j\omega)| = \frac{1}{4} \text{ and occurs at } \omega_0 = 10 \text{ M r/s}$$

The bandwidth, β, of the filter is 500k r/s.

$$|H(j\omega)| = 0 \text{ at } \omega = 0 \qquad \text{and} \qquad \lim_{\omega\to\infty} |H(j\omega)| = 0$$

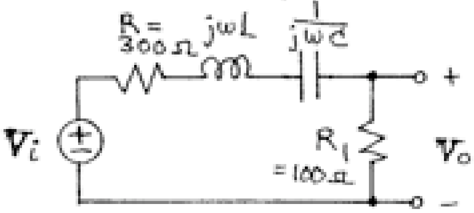
SOL'N: To achieve the peak at ω_0, we may use a series LC in the top rail or a parallel LC from the top to bottom rail. To achieve a gain of 1/4 at ω_0, we must use a vertical resistance to form a V-divider with R.



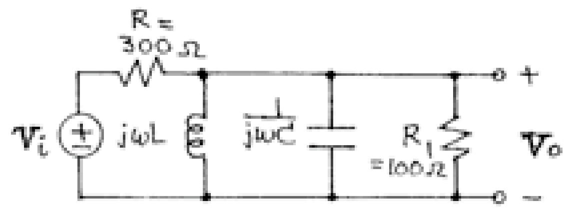
$$\frac{R_1}{R+R_1} = \frac{1}{4}, \quad 3R_1 = R = 300\Omega$$

$$R_1 = 100\Omega$$

We have two possible circuit configurations:



Series configuration



Parallel Configuration

Note: R_1 must be to the right of L and C in order to have any effect from the L and C in the <u>series</u> configuration.

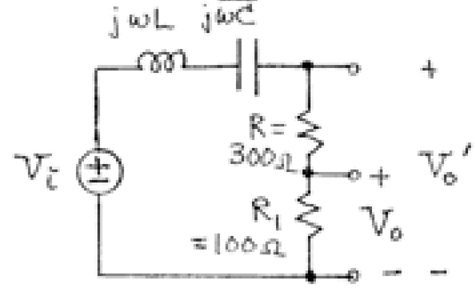
For the series configuration, the L and C are to act like a wire at $\omega_o = 10\text{ M r/s}$.

$$j\omega_o L + \frac{1}{j\omega_o C} = 0 \quad \text{or} \quad \omega_o^2 = (10\text{M r/s})^2 = \frac{1}{LC}$$

The bandwidth when using a series L and C is

$$\beta = \frac{R_{eq}}{L} = 500\text{ k r/s}$$

To determine the value of R_{eq}, we view the filter as a standard RLC filter and a V-divider:



$$H(j\omega) = \underbrace{\frac{V_o'}{V_i}}_{H'} \cdot \frac{V_o}{V_o'}$$

$$H(j\omega) = H'(j\omega) \cdot \frac{R_1}{R + R_1}$$

The cutoff frequencies for $H(j\omega)$ are the same as the cutoff frequencies for $H'(j\omega)$:

$$\omega_{c1,2} = \pm \frac{R_{eq}}{2L} + \sqrt{\left(\frac{R_{eq}}{2L}\right)^2 + \omega_o^2}, \quad \beta = \frac{R_{eq}}{L}$$

where $R_{eq} = R + R_1 = 400\,\Omega$

Using $\beta = R_{eq}/L$, we find L:

$$L = \frac{R_{eq}}{\beta} = \frac{400\,\Omega}{500\,kr/s} = 0.8\text{ mH or } 800\,\mu H$$

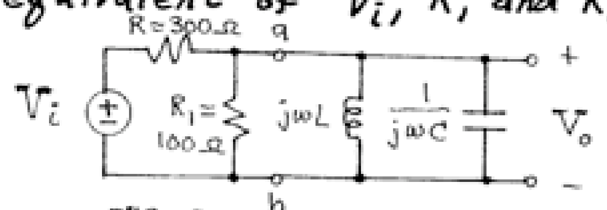
Using $\omega_o^2 = \frac{1}{LC}$ and $L = 800\,\mu H$, we find C:

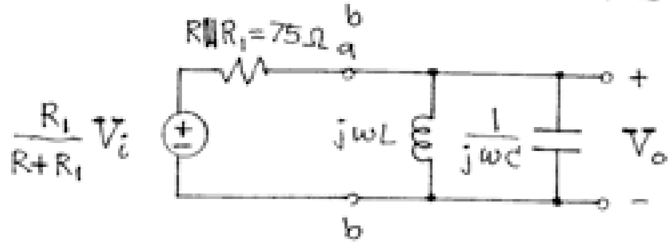
$$C = \frac{1}{\omega_o^2 L} = \frac{1\quad F}{10M \cdot 10\cancel{M} \cdot 800\cancel{\mu}} = \frac{1\,\mu F}{80K}$$

$$C = 12.5\,pF$$

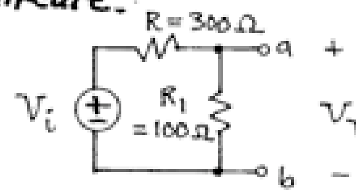
Summary of series RLC: $R_1 = 100\,\Omega$, $L = 800\,\mu H$, $C = 12.5\ F$

For the parallel configuration, we move R_1 to the left of L and C and use a Thevenin equivalent of V_i, R, and R_1.





To find the Thevenin equivalent, we find V_{Th} by finding the open-circuit output V of the V_i, R, and R_1 circuit.



$$V_{Th} = V_i \frac{R_1}{R_1 + R} = \frac{V_i}{4}$$

To find R_{Th}, we turn off V_i and look in from terminals a and b. The resistance seen is

$$R_{Th} = R \,\|\, R_1 = 300\,\Omega \,\|\, 100\,\Omega = 75\,\Omega$$

Using the filter with the Thevenin equivalent, we have

$$H(j\omega) = \frac{V_o}{V_i} = \frac{R_1}{R+R_1}\frac{V_o}{V_i'} = \frac{1}{4} H'(j\omega)$$

$$\text{where } H'(j\omega) \equiv \frac{V_o}{V_i'}, \quad \text{where } V_i' \equiv \frac{R_1}{R+R_1} V_i$$

The cutoff frequencies of $H'(j\omega)$ are the same as the cutoff frequencies of $H(j\omega)$.

$$\omega_{c1,2} = \pm \frac{1}{2R_{Th}C} + \sqrt{\left(\frac{1}{2R_{Th}C}\right)^2 + \omega_o^2}, \quad \beta = \frac{1}{R_{Th}C}$$

$$\omega_o^2 = \frac{1}{LC}$$

Using R_{Th} and β, we find C:

$$C = \frac{1}{R_{Th}\beta} = \frac{1}{75\,\Omega \cdot 500\text{k r/s}} = 26.6\text{ nF}$$

Using L and ω_o^2, we find L:

$$L = \frac{1}{\omega_o^2 C} = \frac{1\text{ H}}{10\text{M}\,10\text{M}\,26.6\text{n}} = 375\text{ nH}$$