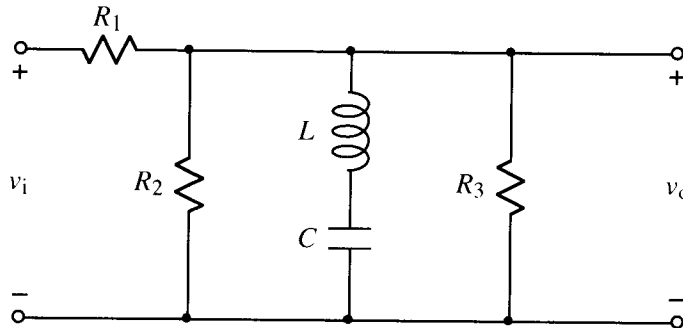


Ex:



$$R_1 = 18 \Omega \quad R_2 = 48 \Omega \quad R_3 = 144 \Omega \quad C = 31.25 \mu\text{F} \quad L = 2 \text{ mH}$$

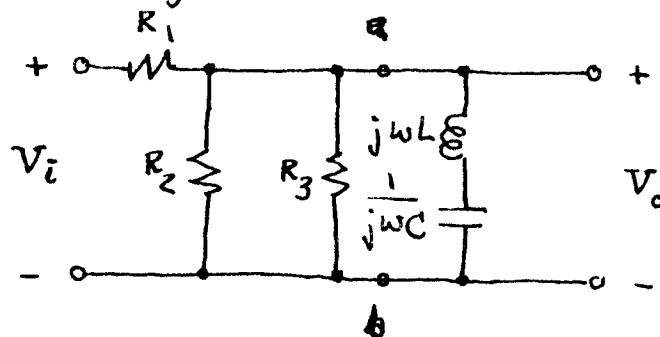
a) What type of filter is the above circuit: a band-pass or a band-reject?

Hint: Use a Thevenin equivalent to combine all the R's into one.

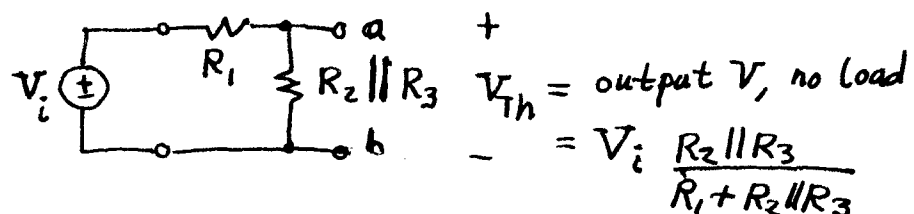
For the filter shown above, calculate the following quantities:

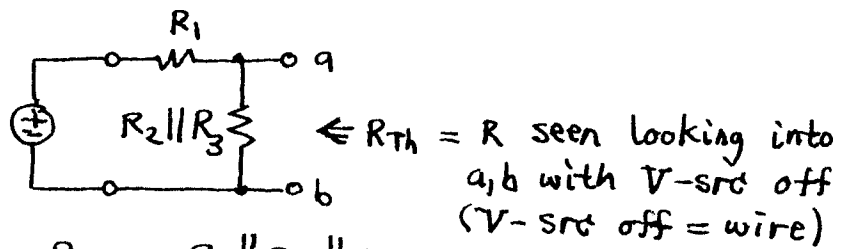
b) ω_0 c) ω_{C1} and ω_{C2} d) β and Q

sol'n: a) The order of the branches between the rails may be changed without affecting the filter's transfer function. Thus, we may use the following schematic:



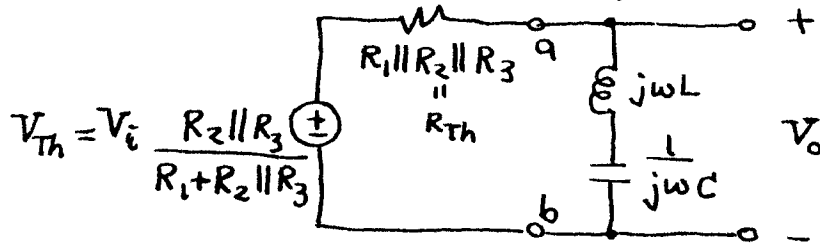
We treat V_i as a V -source and take the Thevenin equivalent to the left of terminals a and b .





$$R_{Th} = R_1 \parallel R_2 \parallel R_3$$

So we have the following picture:



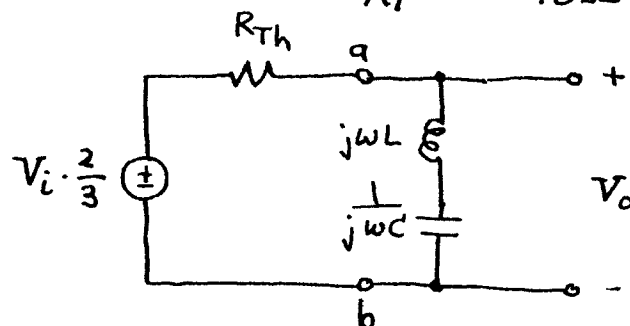
Note: $\frac{R_2 \parallel R_3}{R_1 + R_2 \parallel R_3} = \frac{R_1 \parallel R_2 \parallel R_3}{R_1} = \frac{R_{Th}}{R_1}$

So $V_{Th} = V_i \frac{R_1 \parallel R_2 \parallel R_3}{R_1}$.

Calculated values: $R_1 \parallel R_2 \parallel R_3 = \frac{1}{\frac{1}{18\Omega} + \frac{1}{48\Omega} + \frac{1}{144\Omega}}$

$$R_1 \parallel R_2 \parallel R_3 = 12\Omega$$

$$\frac{R_1 \parallel R_2 \parallel R_3}{R_1} = \frac{12\Omega}{18\Omega} = \frac{2}{3}$$



We effectively have a 2-stage system. The first stage multiplies V_i by $\frac{2}{3}$, and the second stage is an RLC filter.

$$H(j\omega) \equiv \frac{V_o}{V_i} = \frac{V_i \left(\frac{2}{3}\right)}{V_i} \cdot \frac{j\omega L + \frac{1}{j\omega C}}{R_{Th} + j\omega L + \frac{1}{j\omega C}}$$

We put this in standard form:

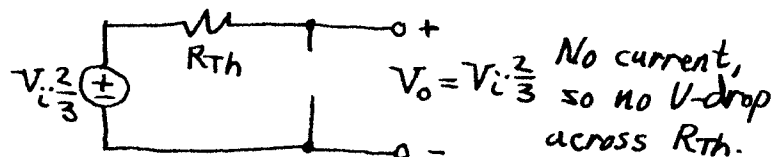
$$H(j\omega) = k \frac{1}{1 \pm jX} \quad \text{where } k = \text{real const} \quad X = \text{real term}$$

$$\text{Here, } k = \frac{2}{3} \quad \text{and} \quad X = \omega L - \frac{1}{\omega C}$$

$$H(j\omega) = \frac{2}{3} \cdot \frac{1}{1 - j \frac{R_{Th}}{\omega L - \frac{1}{\omega C}}}$$

We will use $H(j\omega)$ below. Returning to the question at hand, however, we are asked to determine the type of filter we have. We do this by considering $\omega=0$, $\omega \rightarrow \infty$, and $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$.

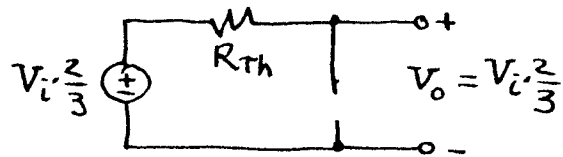
$$\omega=0: \quad j\omega L = j0\Omega = \text{wire} \\ \frac{1}{j\omega C} = \frac{1}{j0\Omega} = -j\infty = \text{open}$$



$$H(j0) = \frac{V_o}{V_i} = \frac{V_i \left(\frac{2}{3}\right)}{V_i} = \frac{2}{3}$$

$$\omega \rightarrow \infty: j\omega L = j\infty = \text{open}$$

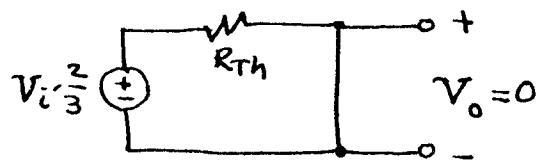
$$\frac{1}{j\omega C} = \frac{1}{j\infty} = -j0 = \text{wire}$$



$$H(j\infty) = \frac{V_o}{V_i} = \frac{V_i (\frac{2}{3})}{V_i} = \frac{2}{3}$$

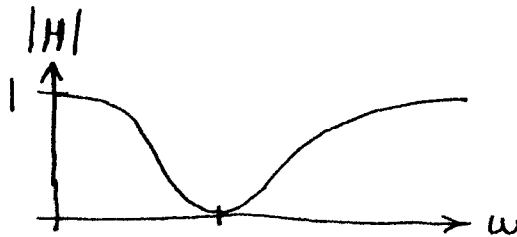
$$\omega = \frac{1}{\sqrt{LC}} = \omega_0: j\omega L = -\frac{1}{j\omega C}$$

$$\text{so } j\omega L + \frac{1}{j\omega C} = 0 \Omega = \text{wire}$$



$$H(j\omega_0) = \frac{0}{V_i} = 0$$

We can now sketch $|H|$ vs ω :



We have a band-reject filter.

$$\begin{aligned}
 \text{b) } \omega_0 &= \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2m(31.25\mu)}} \text{ r/s} \\
 &= \frac{1}{\sqrt{62.5n}} \text{ r/s} = \frac{1}{\sqrt{250\mu}^2} \text{ r/s} \\
 &= \frac{1}{250\mu} = 4k \text{ r/s}
 \end{aligned}$$

$$\text{c) } \omega_{c1,2} \text{ are where } |H(j\omega)| = \frac{1}{\sqrt{2}} \max_{\omega} |H(j\omega)|$$

$$\text{Here, } H(j\omega) = \frac{2}{3} \cdot \frac{1}{1 - j \frac{R_{Th}}{\omega L - \frac{1}{\omega C}}}$$

$$\begin{aligned}
 |H(j\omega)| &= \frac{2}{3} \cdot \frac{1}{\left| 1 + j \frac{R_{Th}}{\omega L - \frac{1}{\omega C}} \right|} \\
 &= \frac{2}{3} \cdot \frac{1}{\sqrt{1^2 + \left(\frac{R_{Th}}{\omega L - \frac{1}{\omega C}} \right)^2}}
 \end{aligned}$$

The smallest value the $\sqrt{\quad}$ can possibly be is $\sqrt{1^2} = 1$. Is it actually this small for any ω ?
 Yes, for $\omega = 0$ or $\omega \rightarrow \infty$ we get

$$\frac{R_{Th}}{\omega L - \frac{1}{\omega C}} = 0, \text{ which we saw}$$

earlier when plotting $|H|$ vs ω .

$$\text{So } \max_{\omega} |H(j\omega)| = \frac{2}{3}.$$

So we want to solve $|H(j\omega)| = \frac{1}{\sqrt{2}} \cdot \frac{2}{3}$:

$$\frac{2}{3} \cdot \frac{1}{\left| 1 + j \frac{R_{Th}}{\omega L - \frac{1}{\omega C}} \right|} = \frac{1}{\sqrt{2}} \cdot \frac{2}{3}$$

$$\text{or } \left| 1 + j \frac{R_{Th}}{\omega L - \frac{1}{\omega C}} \right| = \sqrt{2}$$

We observe that $|1 \pm j| = \sqrt{2}$. So we want

$$\frac{R_{Th}}{\omega L - \frac{1}{\omega C}} = \pm 1$$

Flip both sides upside-down:

$$\frac{\omega L - \frac{1}{\omega C}}{R_{Th}} = \pm 1$$

or

$$\omega L - \frac{1}{\omega C} = \pm R_{Th}$$

or

$$\omega L \pm R_{Th} - \frac{1}{\omega C} = 0$$

or

$$\omega^2 L \pm \omega R_{Th} - \frac{1}{C} = 0$$

or

$$\omega^2 \pm \omega \frac{R_{Th}}{L} - \frac{1}{LC} = 0$$

or

$$\omega_{c1,2} = \frac{\pm \frac{1}{R_{Th}}}{2/L} \pm \sqrt{\left(\frac{1}{2L} R_{Th}\right)^2 + \frac{1}{LC}}$$

Use roots where $\omega_{c1,2} > 0$.

The $\sqrt{\quad}$ term must be larger than

$\frac{R}{2L}$ since the term inside the

$\sqrt{\quad}$ is at least as big as $\left(\frac{R}{2L}\right)^2$.

So we use $+\sqrt{\quad}$ terms:

$$\omega_{c1,2} = \pm \frac{R_{Th}}{2L} + \sqrt{\left(\frac{R_{Th}}{2L}\right)^2 + \frac{1}{LC}}$$

$$\text{Values: } \frac{R_{Th}}{2L} = \frac{12\Omega}{2 \cdot 2mH} = 3k \text{ r/s}$$

$$\frac{1}{LC} = (4k \text{ r/s})^2$$

$$\omega_{c1,2} = \pm 3k + \sqrt{(3k)^2 + (4k)^2} \text{ r/s}$$

$$\omega_{c1} = 2k \text{ r/s}$$

$$\omega_{c2} = 8k \text{ r/s}$$

$$d) \beta \equiv \omega_{c2} - \omega_{c1} = \frac{R_{Th}}{2L} + \sqrt{\quad} - \left(-\frac{R_{Th}}{2L} + \sqrt{\quad}\right)$$

$$\text{or } \beta = \frac{R_{Th}}{L}$$

$$\beta = \frac{12\Omega}{2mH} = 6k \text{ r/s}$$

$$Q \equiv \frac{\omega_0}{\beta} = \frac{4k \text{ r/s}}{6k \text{ r/s}} = \frac{2}{3} \text{ (unitless)}$$

$$\text{Note: } \omega_0 = \sqrt{\omega_{c1} \omega_{c2}} = \sqrt{2k \cdot 8k} \text{ r/s}$$

$$\omega_0 = 4k \text{ r/s (sometimes useful)}$$