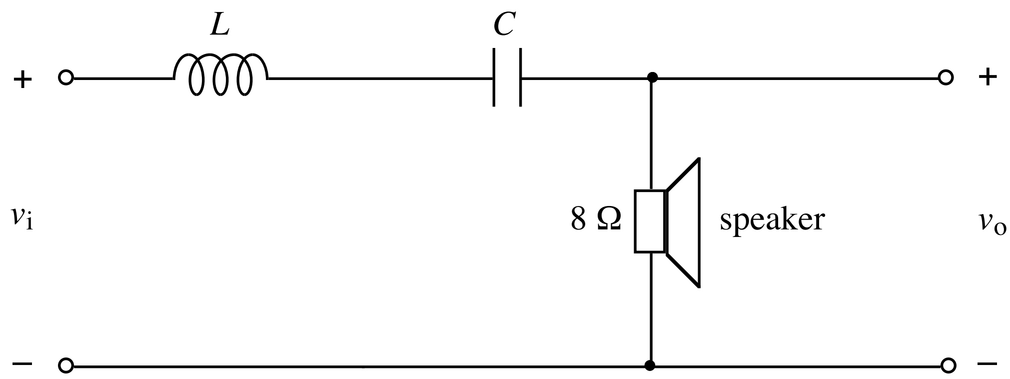
HOMEWORK #3 prob 4 solution

Ex:



The above circuit is part of a simple crossover network for driving a midrange speaker having an impedance of 8Ω. The circuit is described at the following web site: http://www.termpro.com/articles/xover2.html. A more in-depth discussion of crossover networks may be found at http://sound.westhost.com/lr-passive.htm.

a) The web site describing the above bandpass filter suggests using cutoff frequencies of f_{C1} = 130 Hz and f_{C2} = 4 kHz. Determine the L and C values that yield these cutoff frequencies.

b) Plot $|V_o/V_i|$ versus ω.

SOL'N: a) This is a standard band-pass filter. The cutoff frequencies for this filter are as follows:

$$\omega_{C1,2} = \pm\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

where R = 8 Ω.

The following observations simplify our calculations:

$$\omega_{C2} - \omega_{C1} = \left(\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right) - \left(-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right)$$

or

$$\omega_{C2} - \omega_{C1} = \frac{R}{L}$$

or

$$L = \frac{R}{\omega_{C2} - \omega_{C1}}$$

Also,

$$\omega_{C1}\omega_{C2} = \left(-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right)\left(\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right)$$

or

$$\omega_{C1}\omega_{C2} = -\left(\frac{R}{2L}\right)^2 + \left(\sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right)^2 = \frac{1}{LC} = \omega_o^2$$

or

$$C = \frac{1}{\omega_{C1}\omega_{C2}L}$$

Now we compute the cutoff frequencies in r/s:

$$\omega_{C1} = 2\pi f_{C1} = 2\pi(130) \text{ r/s} = 817 \text{ r/s}$$

$$\omega_{C2} = 2\pi f_{C2} = 2\pi(4 \text{ k}) \text{ r/s} = 25.1 \text{ kr/s}$$

Using our formulas from above yields the following:

$$L = \frac{R}{\omega_{C2} - \omega_{C1}} = \frac{8}{25.1\text{k} - 0.817\text{k}} \approx 330 \ \mu\text{H}$$

and

$$C = \frac{1}{\omega_{C1}\omega_{C2}L} = \frac{1}{0.817\text{k} \cdot 25.1\text{k} \cdot 330\mu}\text{F} \approx 150 \ \mu\text{F}$$

b) The circuit is a voltage divider:

$$H(j\omega) = \frac{V_o}{V_i} = \frac{R}{R + j\omega L + 1/j\omega C} = \frac{1}{1 + j\frac{1}{R}(\omega L - 1/\omega C)}$$

We use the following Matlab code to plot the frequency response:

```
% ECE2260F07_HW3p3Matlab.m
%
% Plot of filter's frequency response curve

figure(1)
omega = 1:30e1:30e3;
s = j * omega;

FilterResp = 1./(1 + j * (1/8)*(omega*330e-6 - 1
./ (omega*150e-6)));

plot(omega,abs(FilterResp))
axis([0, max(omega), 0, 1])
xlabel('omega')
ylabel('|H|')
```