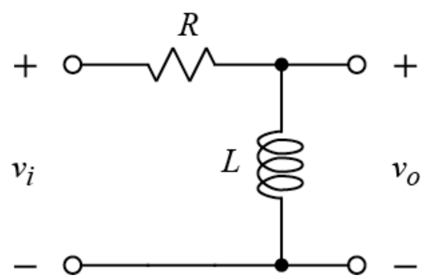
HOMEWORK #3 prob 1 solution

Ex:



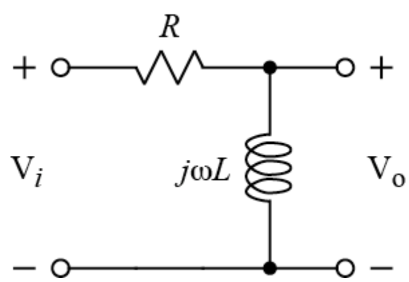
$R = 20\ \text{k}\Omega$ $\qquad L = 200\ \text{nH}$

a) Determine the transfer function V_o/V_i.

b) Plot $|H(j\omega)| \equiv |V_o / V_i|$ versus ω.

c) Find the value of ω where $|\text{Re}(H(j\omega))| = |\text{Im}(H(j\omega))|$.

SOL'N: a) We transform the circuit to the frequency domain.



The voltage-divider formula gives the transfer function, starting with the formula for V_o:

$$V_o = V_i \frac{j\omega L}{R + j\omega L}$$

Dividing by V_i gives the transfer function:

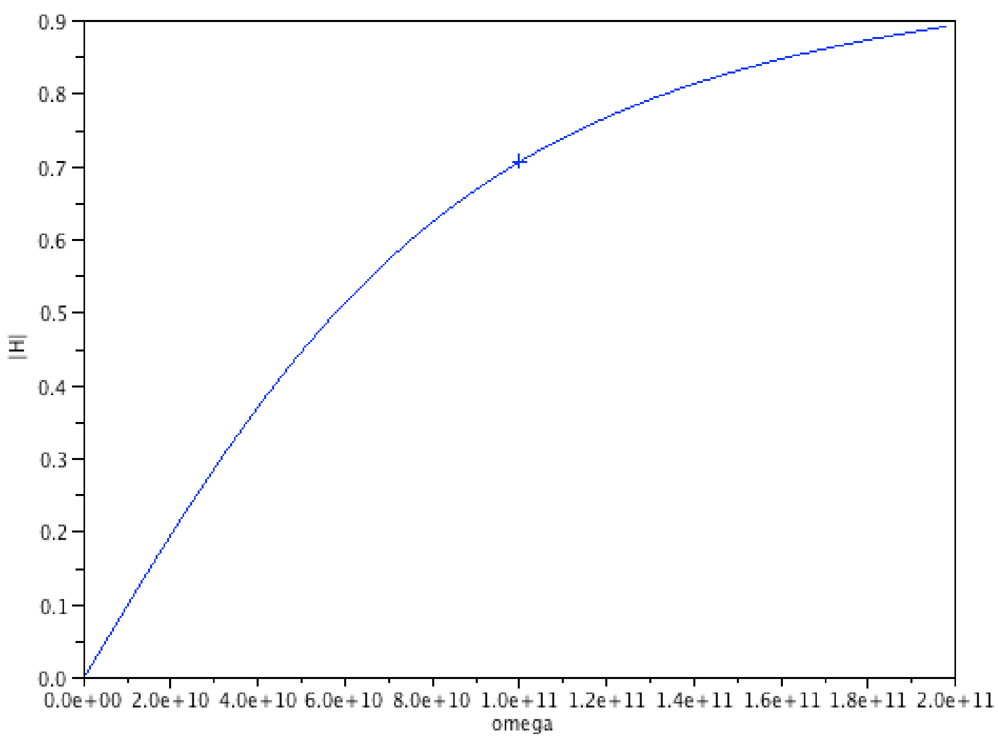
$$H(j\omega) = \frac{V_o}{V_i} = \frac{j\omega L}{R + j\omega L}$$

A better form is obtained by dividing top and bottom by $j\omega L$:

$$H(j\omega) = \frac{1}{1 + \dfrac{R}{j\omega L}} = \frac{1}{1 - j\dfrac{R}{\omega L}} = \frac{1}{1 - j\dfrac{20\text{k}}{\omega 200\text{n}}} = \frac{1}{1 - j\dfrac{100\text{G}}{\omega}}$$

b) The plot is generated with the following SciLab code. (SciLab is open source software.)

```
// ECE2260F11_HW3p1soln.sce
//
// Plot of transfer function of RL high-pass filter.

j = %i  // for complex numbers

R = 20e3;
L = 200e-9;

omega = 1e6:2e9:200e9;
H = 1 ./ (1 - j*R./(omega*L));

omegaC = 100e9;  // plot cutoff freq
Hc = 1 ./ (1 - j*R./(omegaC*L));


plot(omega,abs(H))
plot(omegaC,abs(Hc),'+')

xlabel('omega')
ylabel('|H|')
```



c) We first rationalize $H(j\omega)$.

$$H(j\omega) = \frac{1}{1 - j\dfrac{100\text{G}}{\omega}} \frac{1 + j\dfrac{100\text{G}}{\omega}}{1 + j\dfrac{100\text{G}}{\omega}} = \frac{1}{1^2 + \left(\dfrac{100\text{G}}{\omega}\right)^2} + j\frac{\dfrac{100\text{G}}{\omega}}{1^2 + \left(\dfrac{100\text{G}}{\omega}\right)^2}$$

The real and imaginary parts are positive, so the magnitudes of the real and imaginary parts are the same as the real and imaginary parts.

$$|\text{Re}[H(j\omega)]| = \frac{1}{1^2 + \left(\dfrac{100\text{G}}{\omega}\right)^2}$$

$$|\text{Im}[H(j\omega)]| = \frac{\dfrac{100\text{G}}{\omega}}{1^2 + \left(\dfrac{100\text{G}}{\omega}\right)^2}$$

Since the denominators of the real and imaginary are the same, we need only equate the numerators of the above expressions.

$$1 = \frac{100\text{G}}{\omega}$$

Solving for ω, we discover that it is the same as the cutoff frequency of the filter.

$$\omega = 100\text{G}$$