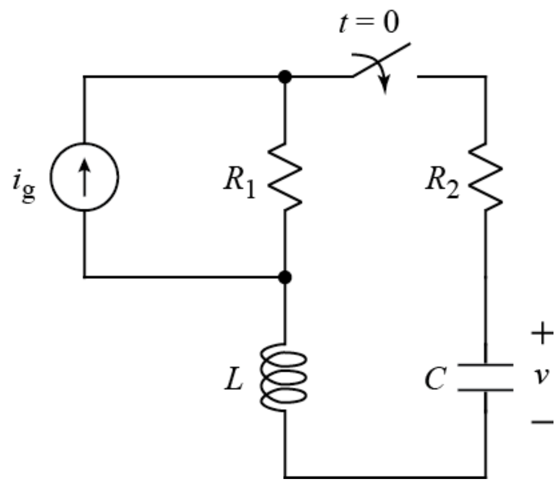
HOMEWORK #2 probs 4 and 5 solution

Ex:



After being open for a long time, the switch closes at $t = 0$.

$i_g = 0.2$ A $R_1 = 50\ \Omega$ $R_2 = 12.5\ \Omega$ $L = 10$ mH $C = 16\ \mu$F

a) State whether $v(t)$ is under-damped, over-damped, or critically-damped.

b) Write a numerical time-domain expression for $v(t)$, $t > 0$, the voltage across C. This expression must not contain any complex numbers.

SOL'N: a) We use the circuit for $t > 0$ to find the characteristic roots. If we convert i_g and R_1 into a Thevenin equivalent, we see ~~that~~ R_1 and R_2 are in series. Since L and C are in series, we have a series RLC.

$$\alpha = \frac{R}{2L} = \frac{50 + 12.5\ \Omega}{2\,(10\text{mH})} = 3.125\ \text{k/s}$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{10\text{mH} \cdot 16\mu\text{F}} = \frac{1\,\text{G}\,r^2/s^2}{160} = \left(\frac{10\text{kr/s}}{4}\right)^2$$

$$s_{1,2} = -3.125 \pm \sqrt{3.125^2 - 2.5^2}\ \text{kr/s} = -5\ \text{kr/s}, -1.25\ \text{kr/s}$$

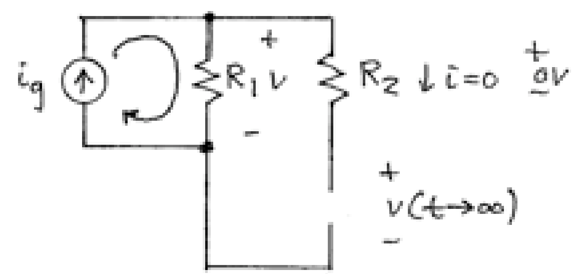
overdamped (real roots)

b) Our form of solution for the overdamped case is

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + A_3 .$$

$$A_3 = v(t \to \infty)$$

For $t \to \infty$, we use L = wire, C = open



$v(t \to \infty)$ equals the voltage across R_1 since no current flows in R_2 (so 0V across R_2) owing to C being an open circuit.

i_g can only flow around the upper left loop. By Ohm's law, the V-drop across R_1 is $i_g R_1$.

$$\therefore v(t \to \infty) = i_g R_1 = A_3$$

or $$A_3 = 0.2A \cdot 50\Omega = 10V$$

Now we match our solution to circuit values at $t = 0^+$:

$$v(0^+) = A_1 + A_2 + A_3 \qquad \left.\frac{dv}{dt}\right|_{t=0^+} = s_1 A_1 + s_2 A_2$$

We consider $t=0^-$ to determine initial values for L and C:

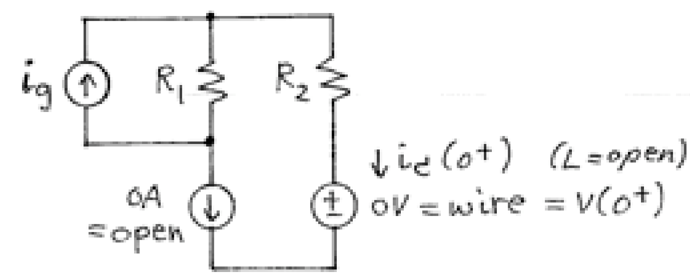
$t=0^-$: L = wire, C = open, switch open

Since the switch is open, no current will flow in L, so we must have $i_L(0^-) = i_L(0^+) = 0A$.

We are given $v(0^+) = 0V$.

At $t=0^+$, we treat L as i-src and C as v-src.

$t=0^+$:



$$v(0^+) = 0V = A_1 + A_2 + A_3 = A_1 + A_2 + 10V$$

$$\left.\frac{dv}{dt}\right|_{t=0^+} = \left.\frac{i_C}{C}\right|_{t=0^+} = 0 = \underset{-5k}{s_1} A_1 + \underset{-1.25k}{s_2} A_2$$

From 2^{nd} eq'n, $A_2 = -4A_1$.
Substitute into 1^{st} eq'n:

$$A_1 - 4A_1 + 10V = 0V, \quad A_1 = \frac{10V}{3}, \; A_2 = \frac{-40V}{3}$$

$$v(t) = \frac{10}{3} e^{-5k/s \cdot t} - \frac{40}{3} e^{-1.25k/s \cdot t} + 10V$$