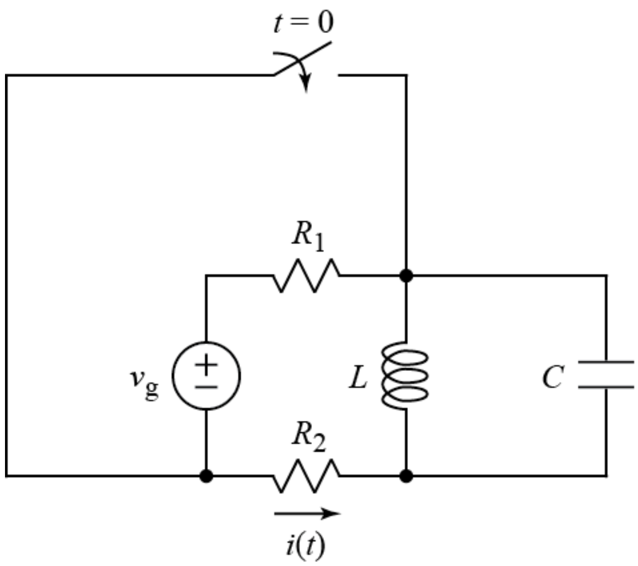
HOMEWORK #2 probs 1 and 2 solution

Ex:



After being open for a long time, the switch closes at $t = 0$.

a) Give expressions for the following in terms of no more than v_g, R_1, R_2 L, and C:

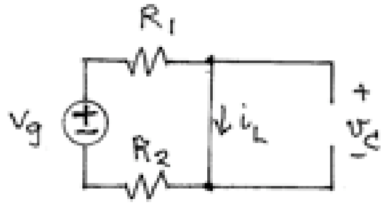
$$i(t = 0^+) \qquad \text{and} \qquad \left.\frac{di(t)}{dt}\right|_{t=0^+}$$

b) Find the numerical value of R_2 given the following information:

$$R_1 = 150\ \Omega \qquad L = 40\text{ mH} \qquad C = 3.2\ \mu\text{F}$$

$$\alpha = 1250\text{ r/s} \qquad \omega_d = 2500\text{ r/s}$$

SOL'N: a) We start with the circuit at $t = 0^-$ to determine the state of the L and C. At $t = 0^-$, L = wire, C = open



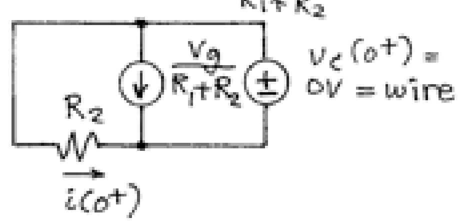
$v_C(0^-) = 0V$ (shorted by L)

$$i_L(0^-) = \frac{v_g}{R_1 + R_2}$$

At $t=0^+$, the switch is closed and $i_L(0^+) = i_L(0^-)$ and $v_C(0^+) = v_C(0^-)$.

V_g and R_1 are shorted out by the switch. This short divides the circuit into two halves that we may solve independently. The one we are interested in consists of R_2, L, and C.

$t=0^+$: $i_L(0^+) = \dfrac{V_g}{R_1+R_2}$ = current source



Because the C acts like a wire, all the current from the L will flow thru the C. So the current in R_2 is zero. (Another way to see this is to observe that R_2 has $0V = V_C$ across it, and by Ohm's law $i(0^+) = 0V/R_2 = 0A$.)

So $i(0^+) = 0\,A$.

For $\left.\dfrac{di}{dt}\right|_{t=0^+}$, we first write $i(t)$

in terms of i_L and/or v_C.

We observe that v_C is across R_2:

$i(t) = v_C / R_2$

Now we differentiate the entire eq'n:

$$\frac{d\,i(t)}{dt} = \frac{1}{R_2}\frac{dv_c}{dt} = \frac{1}{R_2}\frac{i_c}{C}$$

(We have used $i_c = C\frac{dv_c}{dt}$ rearranged.)

We evaluate i_c at $t=0^+$. We noted earlier that all the current from the L goes thru C. If we follow the direction of the current arrow for the L down, over, and up thru the C, we see that

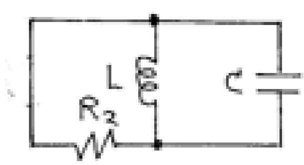
$$i_c(0^+) = -i_L(0^+) = \frac{-V_g}{R_1+R_2}.$$

Thus, $\left.\frac{d\,i(t)}{dt}\right|_{t=0^+} = \frac{1}{R_2 C}\, i_c(0^+)$

or

$$\left.\frac{d\,i(t)}{dt}\right|_{t=0^+} = \frac{-V_g}{R_2 C\,(R_1+R_2)}.$$

b) We use the circuit for $t>0$.



We have a parallel RLC.

$$\alpha = \frac{1}{2R_2C},\quad \omega_d = \sqrt{\omega_0^2-\alpha^2},\quad \omega_0^2 = \frac{1}{LC}$$

Use α: $R_2 = \frac{1}{2\alpha C} = \frac{1}{2(1250)3.2\mu}\,\Omega = 125\,\Omega$