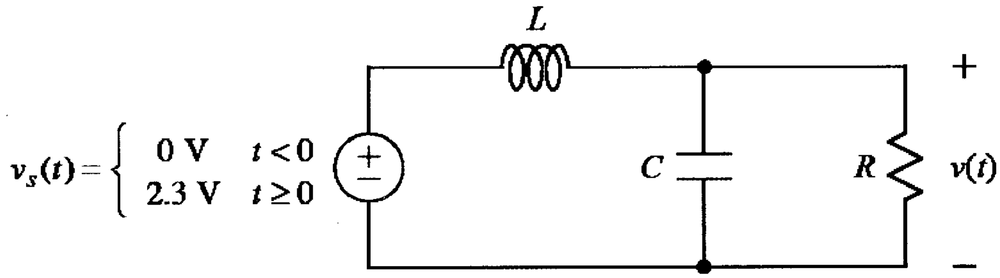
ECE 2260
F11

HOMEWORK #1 prob 3 solution

Ex:



The above circuit models a digital logic gate driving another digital logic gate (modeled as R and C) via a long path of metal on-chip (modeled as L).

L = 10 pH $\quad$ C = 0,4 pF $\quad$ R = 1 GΩ

Find the shape of $v(t)$ versus time. That is, make a rough sketch of $v(t)$ based on the characteristic roots and final value of $v(t)$.

sol'n: We have a parallel RLC, which may be seen by turning off the v-src, which becomes a wire.

Thus, $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(1G)(0.4p)} = \dfrac{1}{0.8\,m}$ /s

$\alpha = 1.25\text{ k/s}$

Characteristic roots:

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

where $\omega_0^2 = \dfrac{1}{LC} = \dfrac{1}{10p(0.4p)}$ r²/s²

or $\omega_0^2 = \left(\dfrac{1}{2p}\right)^2$ r²/s²

or $\omega_0^2 = (500M)^2$

Thus, $s_{1,2} = -1.25k \pm \sqrt{(1.25k)^2 - (500M)^2}$.

Since $\omega_0 \gg \alpha$, the circuit is very underdamped.

Our damping frequency is

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} \approx \sqrt{\omega_0^2} \quad \text{since } \omega_0 \gg \alpha$$

or

$$\omega_d \approx \omega_0 = (500M)^2$$

So our roots are $s_{1,2} \approx -1.25 \pm j500M$ r/s.

Our roots are complex, so the sol'n oscillates at frequency $\omega_d = 500M$.

The frequency of oscillation is much higher than α, so the sol'n will oscillate many times before they decay away as $e^{-\alpha t}$. The sol'n form shows the oscillation and decay:

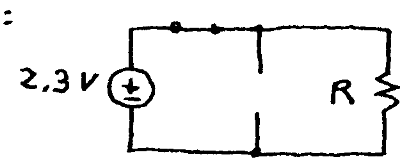
$$v(t) = A_1 e^{-\alpha t} \cos(\omega_d t) + A_2 e^{-\alpha t} \sin(\omega_d t) + A_3$$

Finally, we find A_3, which is the final value of $v(t)$.

$$A_3 = v(t \to \infty)$$

As $t \to \infty$, the circuit stabilizes, and the L acts like a wire and C acts like an open.

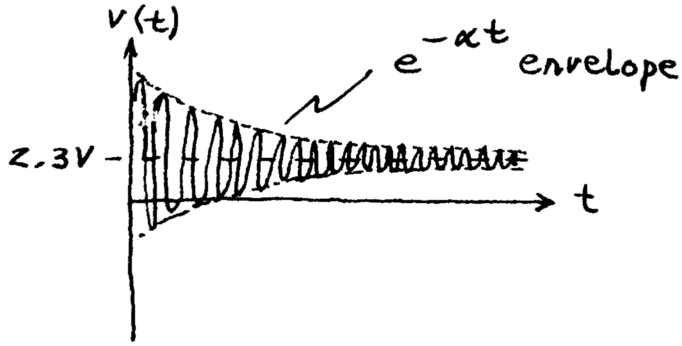
$t \to \infty$:



$+$ $v(t \to \infty) = 2.3V$ $-$

The R must drop the 2.3V.

Our sketch of the sol'n captures the qualitative nature of the sol'n, but it will not capture the initial value of $v(t)$, and it will not capture the size of the oscillations.



Note: the oscillations are actually much more rapid than shown.