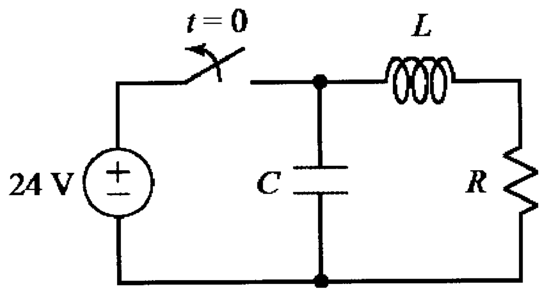
# HOMEWORK #1 prob 2 solution

Ex:




$C = 1$ mF $\qquad L = 25$ mH

A relay is driven by a 24 V power supply, as shown above. Power is turned off at $t = 0$. The current, $i(t)$, for $t > 0$ has two terms that decay exponentially without oscillation. One term dies out quickly, and the other term dies out with a time constant of $\tau = 10$ ms, as in $e^{-t/10\text{ms}}$. Given the time constant and the information in the diagram above, find the value of $R$.

sol'n: From what we are told, we have one characteristic root equal to $-1/10$ms.

Since the circuit is a series RLC, we use $\alpha = R/(2L)$.

As always, $\omega_0^2 = 1/(LC)$.

Our characteristic roots are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

The problem states that the term for the root equal to $-1/10$ms dies out more slowly. This means the magnitude of this root is smaller. This is, therefore, the root corresponding to

$$s = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

So we have $s = -\alpha + \sqrt{\alpha^2 + \omega_0^2} = -1/10\text{ms}.$

We solve for R.

$$-\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}=-\frac{1}{10ms}=-100$$

We get the $\sqrt{\ }$ by itself on one side by adding $\frac{R}{2L}$ to both sides.

$$\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}=-100+\frac{R}{2L}$$

We now square both sides.

$$\cancel{\left(\frac{R}{2L}\right)^2}-\frac{1}{LC}=+100^2-2(100)\frac{R}{2L}+\cancel{\left(\frac{R}{2L}\right)^2}$$

$$\text{where}\quad \frac{1}{LC}=\frac{1}{25m\ 1m}=\left(\frac{1}{5m}\right)^2\ (r/s)^2$$

or

$$\frac{1}{LC}=200^2\ r^2/s^2$$

We rearrange to get R by itself on one side.

$$200\frac{R}{2L}=100^2+200^2=50k\ r^2/s^2$$

$$R=50k\cdot\frac{2L}{200}\quad \Omega/H$$

$$R=\frac{50k}{200}\cdot 2(25m)=\frac{25}{2}\ \Omega$$

or

$$R=12.5\ \Omega$$