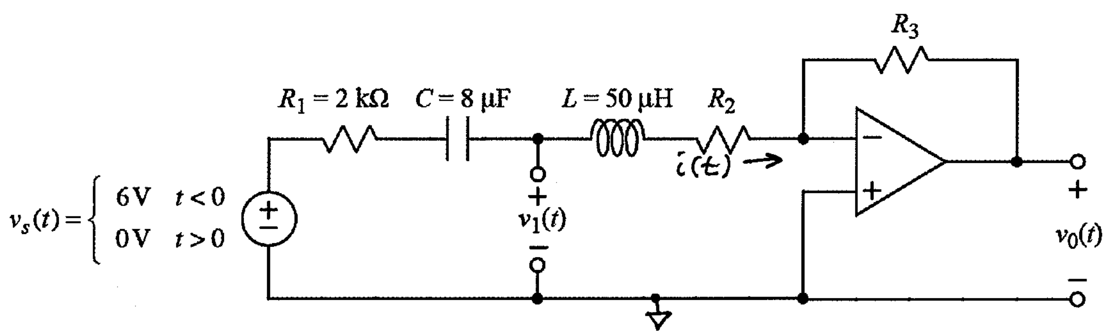
Ex:



The voltage source in the above circuit is off for $t > 0$.

a) Find a symbolic expression for the Laplace-transformed output, $\mathbf{V}_o(s)$, in terms of not more than R_1, R_2, R_3, L, C, and values of sources or constants.

b) Choose a numerical value for R_2 to make

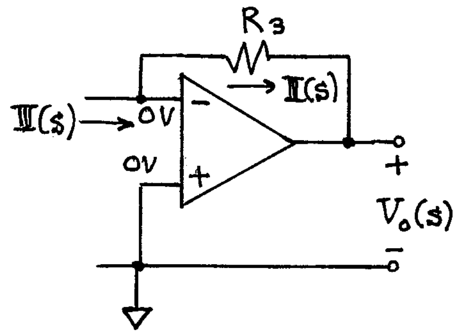
$$v_1(t) = v_m e^{-\alpha t}\cos(\beta t)$$

where v_m, α, and β are real-valued constants.

sol'n: a) No current flows into the op-amp inputs. Thus, current flowing toward the − input from the left will flow through R_3 (and into the op-amp then into the + or − power supply connections to the op-amp [which are not shown] then through the + or − power supply and back to the reference on the bottom rail [which was not shown]).

Since the op-amp has negative feedback, we expect that $v_- = v_+$ at the op-amp inputs. In other words, $v_- = v_+ = 0\,V$, and we have a virtual ground (or reference) at the − input of the op-amp.

We can find $V_o(s)$ from current, $\mathbb{I}(s)$, flowing toward the − input of the op-amp.

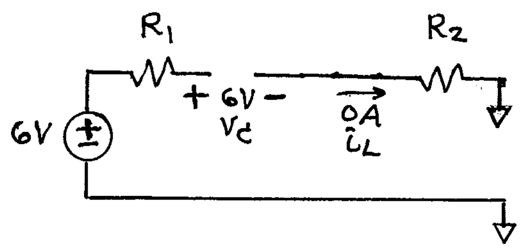


We have $V_o(s) = -I(s)\,R_3$.

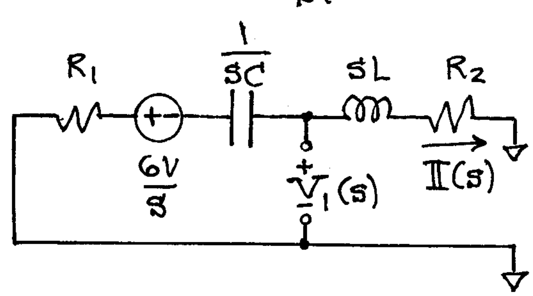
To find $I(s)$, we may treat the − input as reference.

First, however, we find initial conditions for the L and C.

$t=0^-$: $v_s(t) = 6V$, $C = \text{open}$, $L = \text{wire}$



We move to $t>0$ and include initial conditions on C. $V_s(t) = 0\,V = \text{wire}$ for $t>0$.



We have $$I(s) = -\frac{6V}{s}\,\frac{1}{sL + R_1 + R_2 + \frac{1}{sC}}$$

or $$\mathbb{I}(s) = \frac{-6V/L}{s^2 + \frac{R_1+R_2}{L} s + \frac{1}{LC}}$$

So $$V_o(s) = -\mathbb{I}(s) R_3$$

$$V_o(s) = \frac{6V\ R_3/L}{s^2 + \frac{R_1+R_2}{L} s + \frac{1}{LC}}$$

b) $V_1(s)$ is the same as the V-drop across L and R_2.

$$V_1(s) = \mathbb{I}(s)(sL + R_2)$$

or
$$V_1(s) = \frac{-(6V/L)(sL + R_2)}{s^2 + \frac{R_1+R_2}{L} s + \frac{1}{LC}}$$

or
$$V_1(s) = -6V \frac{s + R_2/L}{s^2 + \frac{R_1+R_2}{L} s + \frac{1}{LC}}$$

From the form of $v_1(t)$ given in the problem, we have another form for $V_1(s)$:

$$V_1(s) = \mathcal{L}\{v_m e^{-\alpha t}\cos(\beta t)\}$$

$$V_1(s) = v_m \frac{s+\alpha}{(s+\alpha)^2 + \beta^2}$$

$$V_1(s) = v_m \frac{s+\alpha}{s^2 + 2\alpha s + \alpha^2 + \beta^2}$$

Matching coefficients of the powers of s in the two forms of $V_1(s)$, we have the following equations:

$$R_2/L = \alpha, \quad \frac{R_1+R_2}{L} = 2\alpha, \quad \frac{1}{LC} = \alpha^2 + \beta^2$$

We have $\frac{R_2}{L} = \alpha = \frac{R_1 + R_2}{2L}$.

The solution is $R_2 = R_1 = 2k\Omega$.

$$R_2 = 2k\Omega$$

Now we consider whether this solution actually works. That is, we determine whether our solution is actually under-damped.

$$\alpha = \frac{R_2}{L} = \frac{2k\Omega}{50\mu H} = 40M \text{ r/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{50\mu H \cdot 8\mu F} = \frac{(1M)^2}{400}$$

So $\alpha^2 - \omega_o^2 = 40^2 M^2 - \frac{1M^2}{400} > 0$.

Thus, we have an over-damped solution. Our characteristic roots will be real. <u>We can't solve the problem!</u>

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

$$= -\alpha \pm \alpha\sqrt{1 - \left(\frac{\omega_o}{\alpha}\right)^2}$$

$$\doteq -\alpha \pm \alpha\left(1 - \frac{1}{2}\left(\frac{\omega_o}{\alpha}\right)\right) \text{ since } \omega_o \ll \alpha$$

$$\doteq -\frac{1}{2}\frac{\omega_o}{\alpha} \text{ and } -2\alpha$$

$$\doteq -\frac{\frac{1}{2}\frac{1M}{20}}{40M} \text{ and } -2(40M) \text{ r/s}$$

$$\doteq -\frac{1}{1600} \text{ and } -80M \text{ r/s}$$

We have one slow decay (that is probably very small in the solution) and one fast decay that is like a time constant $L/(R_1+R_2)$.