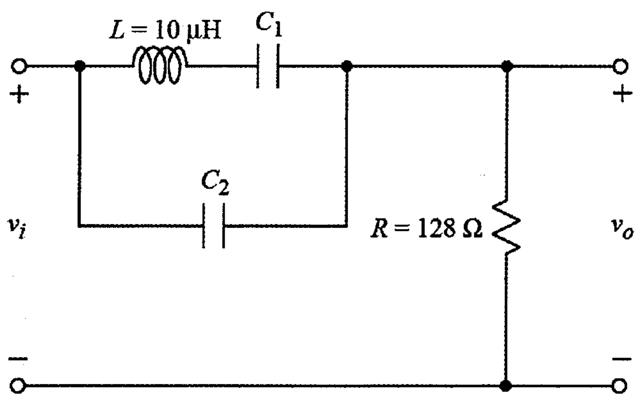
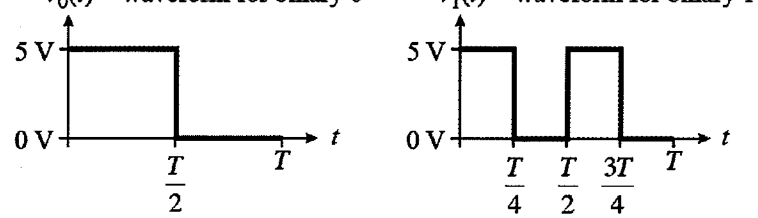
ECE 2260
F 11

FINAL EXAM prob 2 solution

Ex:



$v_0(t)$ = waveform for binary 0 $v_1(t)$ = waveform for binary 1



T = one period of $v_0(t)$ = 6.28 μs

$$v_0(t) = \begin{cases} 5\text{ V} & 0 \le t < T/2 \\ 0\text{ V} & T/2 \le t < T \end{cases} \qquad v_1(t) = \begin{cases} 5\text{ V} & 0 \le t < T/4 \text{ and } T/2 \le t < 3T/4 \\ 0\text{ V} & T/4 \le t < T/2 \text{ and } 3T/4 \le t < T \end{cases}$$

The above filter circuit is being used in a communication system to detect whether received signals represent binary zeros or binary ones. The waveforms are designed to be detected using a method that detects whether the signal is high or low in each segment of length $T/4$, but only the analog filter circuit shown above is available.

The plan is to use the filter shown above to detect waveforms for zeros by designing the filter to pass the fundamental frequency, ω_0, of $v_0(t)$ but block the fundamental frequency, $2\omega_0$, of $v_1(t)$. (Presumably a different filter would be designed to detect waveforms for ones, but that filter is of no concern here.)

a) Find values of $C_1 \neq 0$ and $C_2 \neq 0$ such that the magnitude of the filter's transfer function, H, equals one for the fundamental frequency, ω_0, of $v_0(t)$ and zero for frequency $2\omega_0$.

b) If $v_1(t)$ is the input signal to the filter, find the magnitude of the $6\omega_0$ signal coming out of the filter, (which is the lowest frequency component of $v_1(t)$ that should get through the filter). Hint: first find a_6 and b_6 for the following Fourier series representation of $v_1(t)$.

$$v_1(t) = a_v + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)$$

sol'n: a) We want $H(j\omega_0) = 1$ and $H(j2\omega_0) = 0$.

We achieve $H=1$ when L and C_1 are at resonance.

$$\omega_0^2 = \frac{1}{LC_1} \quad \text{or} \quad C_1 = \frac{1}{L\omega_0^2}$$

The fundamental frequency is $\omega_0 = \frac{2\pi}{T}$

or $\omega_0 = \frac{2\pi}{6.28\,\mu s} \approx 1\,M\,r/s.$

So $C_1 = \frac{1}{10\mu \cdot (1M)^2}\,F = \frac{1}{10}\,\mu F$ or $100\,nF$.

We achieve $H=0$ when $\frac{1}{j\omega C_2} \parallel \left(\frac{1}{j\omega C_1} + j\omega L\right) = \infty$.

This means we want the denominator of the parallel impedance (product over sum) to be zero.

Thus, $\frac{1}{j\omega C_2} + \frac{1}{j\omega C_1} + j\omega L = 0$ @ $\omega = 2\omega_0$.

$$\frac{1}{j2\omega_0 C_2} = -\frac{1}{j2\omega_0 C_1} - j2\omega_0 L$$

$$\text{or} \quad j2\omega_0 C_2 = \frac{1}{-\frac{1}{j2\omega_0 C_1} - j2\omega_0 L}$$

$$C_2 = \frac{1}{-\frac{j2\omega_0}{j2\omega_0 C_1} - (j2\omega_0)^2 L}$$

$$C_2 = \frac{1}{-\frac{1}{C_1} + 4\omega_0^2 L}$$

$$C_2 = \frac{1}{-\frac{1}{100n} + 4(1M)^2 \cdot 10\mu} \, F$$

$$C_2 = \frac{1}{-10M + 40M} \, F$$

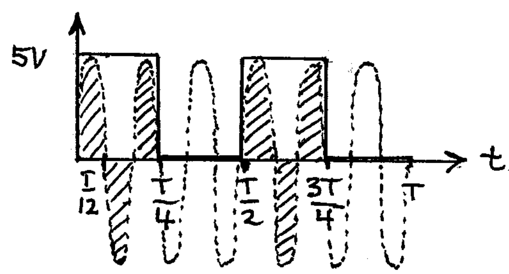
$$C_2 = \frac{1}{30M} \, F = \frac{1}{30} \mu F \text{ or } 33.3\, nF$$

b) We have $a_6 = \frac{2}{T}\int_0^T v_i(t)\cos(6\omega_0 t)\,dt$

$$b_6 = \frac{2}{T}\int_0^T v_i(t)\sin(6\omega_0 t)\,dt$$

We may observe that $v_i(t) = 2.5V + \text{odd func.}$
This means $a_6 = 0$.

A picture of $v_i(t)\cdot\cos(6\omega_0 t)$ also shows that the $\int_0^T v_i(t)\cos(6\omega_0 t)\,dt = 0$, since the integral equals the area under the product curve.



The above picture shows the product of $v_i(t)$ and $\sin(6\omega_0 t)$ that gives the value of b_6.

We have some cancellation of areas, but we are left with two positive lobes.

$$b_6 = 2\left(\frac{2}{T}\right)\int_0^{T/12} 5V \sin\left(6\cdot\frac{2\pi}{T}t\right)dt$$

$$b_6 = \left(\frac{20}{T}\right) \left. \frac{-\cos\left(\frac{12\pi}{T} t\right)}{\frac{12\pi}{T}} \right|_0^{T/12} \text{ V}$$

or

$$b_6 = \frac{5}{3\pi} (--1 - -1) \text{ V}$$

or

$$b_6 = \frac{10}{3\pi} \text{ V} = 1.06\text{V}$$

Now we calculate $H(j6\omega_o)$.

$$H(j6\omega_o) = \frac{R}{\frac{1}{j6\omega_o C_2} \,\Big\|\, \left(\frac{1}{j6\omega_o C_1} + j6\omega_o L\right) + R}$$

$$= \frac{128}{\frac{1}{j6M \cdot \frac{1}{30}\mu} \,\Big\|\, \left(\frac{1}{j6M \cdot 100n} + j6M \cdot 10\mu\right) + 128}$$

$$= \frac{128}{-j5 \,\|\, \left(-j\frac{5}{3} + j60\right) + 128}$$

$$= \frac{128}{128 + -j\frac{15}{3} \,\|\, j\frac{175}{3}} = \frac{128}{128 + -j\frac{5}{3} \cdot 3 \,\|\, 35} = \frac{128}{128 - j\frac{165}{38}}$$

$$H(j6\omega_o) \doteq 1$$

$$|v_{o6}(t)| = |H(j6\omega_o)|\,|b_6| \doteq 1.06\text{V} \approx \left(\frac{10}{3\pi}\right) \text{V}$$

$$|v_{o6}(t)| \doteq 1.06 \text{ V}$$